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# Using Regression Analysis of Observed Licenses to Calculate a Reasonable Royalty for Patent Infringement

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Nobel laureate Friedrich Hayek saw the principal role of markets not as the allocation of scarce goods, but rather the revelation and communication of information about the value of those goods.[1] Patents, like any assets, are sold or licensed with some inherent uncertainty about their value.[2] However, as a patent is sold or licensed, that uncertainty diminishes as each new price struck at arm's length reveals information about what parties are willing to pay to gain access to the technology and what the technology's owner is willing to accept to grant that access. Some uncertainty occurs within product markets, as the demand for a patented technology among consumers might not be observable at the time of licensing. Alternatively, uncertainty can exist with respect to the validity and scope of a patent itself, separate from the demand for the technology that the patent purportedly covers. That is, the probability that its owner will be able to enforce a patent against a particular infringer is less than certain. The patent-granting process is imperfect, and not every patent granted would be found valid by a judge or jury in an eventual infringement trial. Similarly, the claim-construction portion of a patent litigation can yield broader or narrower claims than expected, which might increase or decrease the probability that a particular product infringes the patent in suit. A negotiated royalty rate observed for a patent will account for the uncertainty that a particular patent is valid and that the patent-practicing products, if unlicensed, would infringe that patent. That is, the royalty will

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1 Friedrich A. Hayek, *The Use of Knowledge in Society*, 35 AM. ECON. REV. 519, 526 (1945).

2 *See* Daniel F. Spulber, *How Patents Provide the Foundation of the Market for Inventions*, 11 J. COMPETITION L. & ECON. 271 (2015).

reflect the market-disciplined probability that the patent can be successfully asserted against one whose products practice the patent.

As there occur events providing new information about a patent's validity or scope (such as claim-construction rulings or *inter partes* review (IPR) before the Patent Trial and Appeal Board (PTAB)), the market will incorporate that newly revealed information into the market's probabilistic valuation of that patent. New license agreements for the patent in suit—negotiated at arm's length between the licensor and willing third parties—will contain royalty rates that reflect the incremental recalibration of that market valuation. Over time, the uncertainty associated with any given patent's value will decrease, all other things being equal. For the purposes of this analysis, I focus on the uncertainty of patent validity and infringement, and, unless otherwise noted, I assume for simplicity that the patent's commercial value is constant. Observing how negotiated royalty rates for the patent in suit change on the margin as validity or infringement information is revealed helps to bridge the gap between that patent's observed values and its hypothetical value on the eve of first infringement, when as a matter of law courts have assumed the patent in suit to be absolutely valid and infringed.

## I. Using the Line of Best Fit Within the *Georgia-Pacific* Framework

The very first *Georgia-Pacific* factor asks the finder of fact to examine licenses for the patent in suit.[3] However, the assumptions of the hypothetical negotiation, under factor 15 of *Georgia-Pacific*, differ from the actual negotiation setting for most observed patent licenses. For example, the Federal Circuit cautions against using license agreements that were executed as only one component of the consideration exchanged in an agreement settling litigation.[4] In addition, license agreements willingly negotiated at arm's length will incorporate into the observed royalty the probability that the patent is valid and infringed—that is, the royalty observed in licenses that the licensor negotiates with third parties will be reduced by some amount to account for the probability that the patent is *not* both valid and infringed. As later rulings reveal the probability with which the patent is valid and infringed, observed

[3] Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970), *mod. and aff'd*, 446 F.2d 295 (2d Cir. 1971).

[4] LaserDynamics, Inc. v. Quanta Comput., Inc., 694 F.3d 51, 77 (Fed. Cir. 2012); Hanson v. Alpine Valley Ski Area, Inc., 718 F.2d 1075, 1078–79 (Fed. Cir. 1983); *see also* Rude v. Westcott, 130 U.S. 152, 164 (1889) ("[A] payment of any sum in settlement of a claim for an alleged infringement cannot be taken as a standard to measure the value of the improvements patented, in determining the damages sustained by the owners of the patent in other cases of infringement. Many considerations other than the value of the improvements patented may induce the payment in such cases."). *But see* ResQNet.com, Inc. v. Lansa, Inc., 594 F.3d. 860, 872 (Fed. Cir. 2010) ("This court observes . . . that the most reliable license in this record arose out of litigation.").

royalties will converge over time on the royalty contemplated in *Georgia-Pacific*'s hypothetical negotiation, which is predicated on the assumption that the patent in suit is absolutely valid and infringed.

### *A. The Line of Best Fit*

An expert economic witness can use well-established statistical techniques to estimate a regression model relating the passage of time and other explanatory factors to the observed royalties that third parties have willingly agreed to pay for the patent in suit in comparable licenses negotiated at arm's length. The regression model will predict the slope and intercept coefficients for a line of best fit, which will identify the relationship between the explanatory variables and the observed royalties that best fits the data. Because later royalties for a given patent are negotiated with more information than earlier royalties for the same patent, one can infer that the trend in the royalty rate over time captures the accretion of information that voluntary market transactions incrementally reveal about the patent's validity and scope. The willing actions of third parties in continuing to license the patented technology are themselves pieces of information indicating incremental changes in beliefs concerning the patent's validity and scope. Therefore, predicting a future royalty on the basis of the line of best fit will yield a royalty that approximates (and, as we shall see, provides a lower bound on) the royalty implied by the hypothetical negotiation's assumptions of validity and infringement.

One can use the line of best fit to predict a voluntarily negotiated royalty at any time during the patent's life, including the royalty emerging from the hypothetical negotiation on the eve of first infringement. The line of best fit is optimally employed to predict a royalty at some time after rulings on challenges to the validity of the patent in suit. Typically, the line of best fit will be most valuable in revealing the royalty that would result in a voluntary, arm's-length hypothetical negotiation between the patent owner and a willing third party at the time of trial. That negotiation would be predicated on the parties' beliefs about the probability that the patent in suit would be found at trial to be valid and infringed. That is to say that the negotiation would *not* proceed on the assumption (imposed by case law in the *Georgia-Pacific* hypothetical negotiation) that the patent in suit is absolutely valid and infringed. If inflation or deflation has occurred over the life of the patent in suit, one must convert the royalty rates derived from the line of best fit to constant dollars to enable an apples-to-apples comparison over time.

### *B. Are the Observed Licenses Sufficiently Similar to the License from the Hypothetical Negotiation?*

A potential complication in predicting royalties for the patent in suit on the basis of observed licenses is ensuring that the comparator licenses are sufficiently similar to the license agreement that would emerge between the licensor and the defendant licensee in the hypothetical negotiation on the eve of the defendant's first infringement. At a minimum, three factors require consideration.

#### *1. Similar Willingness to Accept*

First, the licensor must have a similar minimum willingness to accept. If the licensor does not compete with its potential licensees, this concern is less significant, as the opportunity cost of licensing a noncompetitor (on a nonexclusive basis) is unlikely to change substantially from license to license. However, if the licensor and the licensee are competitors, the observed licenses for differently situated licensees might reflect the licensor's varying opportunity cost of licensing, which in turn will depend on the degree of consumer substitutability between the licensor's product and the licensee's product. For a licensor that competes with its licensee, the expert economic witness must include in the analysis only license agreements that are similar to the hypothetically negotiated license agreement.

If there is a large enough sample of observed licenses (providing enough degrees of freedom in the statistical sense), an alternative methodology might be possible: the expert witness can include additional variables in the regression that identify the effects of differently situated licensees or other differences between observed licenses for the patent in suit and the hypothetical negotiation. A simple example of including an additional variable is including a "dummy variable" to measure whether a firm is a horizontal competitor. (A dummy variable takes the value of 1 if a qualitative assessment is true (in this case, whether the licensee is a horizontal competitor of the patent holder) and a value of 0 if that assessment is false.[5]) If richer data exist, the expert could even replace that dummy variable with a variable that continuously measures the degree of competition, such as a diversion ratio. However, if too few licenses exist to enable that identification, the expert might need to exclude from the analysis those licenses for which statistical adjustments are not possible. As a practical matter, in many litigated cases, a sufficient number of licenses for this kind of econometric analysis will not exist.

[5] *See, e.g.*, James H. Stock & Mark W. Watson, Introduction to Econometrics 153 (Addison-Wesley 3d ed. 2011).

### *2. Similar Available Outside Alternatives*

Second, the licensees should have similar available outside alternatives to the patented technology. The available outside alternatives will help determine each licensee's maximum willingness to pay. When noninfringing alternatives to the patent in suit vary across potential licensees, the expert economic witness must include only licenses with sufficiently similarly situated licensees.[6] If those alternatives vary across licensees, the expert must account for how those differences affect a reasonable royalty.

Nonetheless, licensees need not be in the same industry as the hypothetical licensee to have a similar maximum willingness to pay. A manufacturer that incorporates a patented technology into its product and a downstream user of that manufactured product might have the same set of available and acceptable noninfringing alternatives to the patent in suit and might have the same maximum willingness to pay for the patented technology.

An additional consideration when the observed licensee and the hypothetical licensee are in different industries or different stages of a vertical production process will be how a royalty payment would affect the profitability of using the patented technologies. The degree to which a licensee would pass a royalty through to consumers or downstream firms will affect how much the demand for the licensed product changes in response to the license. The effect will also vary across royalty structures. Those considerations strongly caution against the use of traditional accounting methodologies, by themselves, for analyzing reasonable royalties, because those methods are ill equipped to answer the economic questions of the own-price and cross-price elasticities of demand that arise when identifying and evaluating similarly situated licensees.

### *3. The Distribution of Bargaining Power*

Third, as I have previously explained elsewhere, the distribution of bargaining power among the licensor and the licensee might vary greatly from one negotiation to another.[7] The measure of the parties' relative bargaining power ultimately is each party's willingness to walk away from the license negotiation. Similarly, bargaining power has been characterized elsewhere in the economic literature by a measure of each party's discount rate.[8] That is, the rate at which a party is willing to trade profits today for greater profits tomorrow helps determine the percentage of the surplus that a party receives from

[6] The expert could also adjust for differences in observable factors that measure a licensee's maximum willingness to pay. However, there might not exist enough data to make that adjustment.

[7] J. Gregory Sidak, *Bargaining Power and Patent Damages*, 19 Stan. Tech. L. Rev. 1, 26–28 (2015).

[8] *See, e.g.*, Ariel Rubinstein, *Perfect Equilibrium in a Bargaining Model*, 50 Econometrica 97 (1982).

the successful negotiation of a license. The party with the lower discount rate can afford to be more patient in the negotiation, all other things being equal.

Factors that affect bargaining power might also affect the licensor's minimum willingness to accept or the licensee's maximum willingness to pay.[9] For example, if a company faces a liquidity crisis, it might have less bargaining power *and* a lower willingness to accept. "Willingness" to accept is a misnomer in this situation; it is more accurate to speak of the licensee's *compulsion to accept*. Consider a company that lacks the liquid assets or credit line even to make its payroll or to make a scheduled payment for a necessary input to production, such as electricity or steel. For that company, time is of the essence in obtaining revenue. Too much patience in a patent-licensing negotiation will imperil that company's ability to continue operating. Likewise, given that the company might shut down absent the cash influx from a license agreement, the company's opportunity cost of agreeing to a license will be low. Given its economic distress, that company will have an artificially suppressed minimum willingness to accept *and* reduced bargaining power.

The expert economic witness should be careful to consider excluding from the analysis any licenses in which the parties' relative bargaining power might differ substantially from the bargaining power of the licensor and the defendant infringer in the hypothetical negotiation, as in the case of the economic duress described above. It is also important to differentiate between a company-wide measure of patience, such as the firm's weighted average cost of capital, and its project-specific discounting and risk. When assessing bargaining power at the time of each license and at the time of the hypothetical negotiation, the expert should use the best available economic evidence of each party's ability to wait.

If a particular license—by its terms or by the circumstances surrounding its negotiation—differs greatly from the hypothetical negotiation in one of the ways described above, that license still might be informative as a bound on a reasonable royalty, perhaps after some adjustments are made. Typically, the expert economic witness is limited by the availability of data. The witness should thus identify as much helpful information as possible from an observed license, even if the license's only contribution is to identify an upper or lower bound on the royalty that would emerge from the hypothetical negotiation. However, where a license differs in multiple dimensions—for example, when a litigation settlement in the presence of widespread infringement includes non-cash consideration, such as a cross license to the licensee's intellectual property—it might not be possible for the expert economic witness to make

[9] *See* Sidak, *Bargaining Power and Patent Damages*, *supra* note 7, at 17–20.

the necessary adjustments and conversions to render a license reliably informative for the task at hand.[10]

Consider the effect of widespread infringement on the terms of a settlement of litigation.[11] Widespread infringement suppresses the licensor's bargaining power (relative to the licensor in the hypothetical negotiation), if not also its willingness to accept. In addition, the licensor might have less bargaining power at the time of settlement if widespread infringement undermines the licensor's ability to monetize its patent rights but does not reduce its costs of developing those rights. Moreover, the *collective* opportunism of those who pervasively infringe the licensor's patent rights will suppress the licensor's bargaining power (relative to the licensor's bargaining power in the hypothetical negotiation) and thus its willingness to accept. A royalty observed at the time of a litigation settlement under the condition of widespread infringement could actually lie below the bargaining range contemplated by the hypothetical negotiation between a willing licensor and a willing licensee.[12]

In such a scenario, prudence counsels against including such a license in the analysis. Typically, an expert economic witness would need to exclude litigation settlement agreements from an analysis of observed licenses, because some elements of the consideration exchanged in the settlement will be unobservable, such that the expert cannot reliably adjust for the effects of those elements. For example, it would be impossible for an expert economic witness in one case to evaluate the private (and confidential) information from the sealed record in another case to determine the value of eliminating the alleged infringer's noninfringement defense as part of a settlement agreement. Because the value of settling litigation rests on unobservable private information, it is virtually impossible for an expert economic witness to identify the value of the settlement's consideration flowing to the patent holder or the infringer.

## II. Bayesian Updating of a Patent's Expected Value

After identifying licenses for the patent in suit that have been executed with third parties under circumstances comparable to the hypothetical negotiation, the expert economic witness must determine the extent to which new

[10] *See* J. Gregory Sidak, *Converting Royalty Payment Structures for Patent Licenses*, 1 Criterion J. on Innovation 901 (2016).

[11] *See* Michael P. Akemann, John A. Blair & David J. Teece, *Patent Enforcement in an Uncertain World: Widespread Infringement and the Paradox of Value for Patented Technologies*, 1 Criterion J. on Innovation 861 (2016).

[12] *See, e.g.*, J. Gregory Sidak, *The Meaning of FRAND, Part I: Royalties*, 9 J. Competition L. & Econ. 931, 938–39 (2013); Sidak, *Bargaining Power and Patent Damages*, *supra* note 7, at 28–29.

increments of information on validity and infringement, such as information gleaned from rulings on validity challenges or claim construction, explain the variation in the specific royalties observed in those comparable third-party licenses.

### *A. Bayes' Theorem and Conditional Probability*

The process by which new information changes valuations by decreasing uncertainty is called Bayesian updating. In probability theory, one uses Bayes' Theorem to calculate the conditional probability that an event occurs (or that some statement is true) given the information that is currently known.[13] A conditional probability is the likelihood that an event occurs *given* the occurrence of some other event or the truth of some other statement. For example, the probability that it snows in a given location will vary with the month of the year. The *unconditional* probability that it snows in Boston on any given day (throughout the year) differs greatly from the *conditional* probability that it snows in Boston given that it is February or given that it is July. This example of a conditional probability illustrates how the probability of an event occurring varies depending on one or more variables. Bayes' Theorem identifies the relationship between conditional probabilities, as follows:

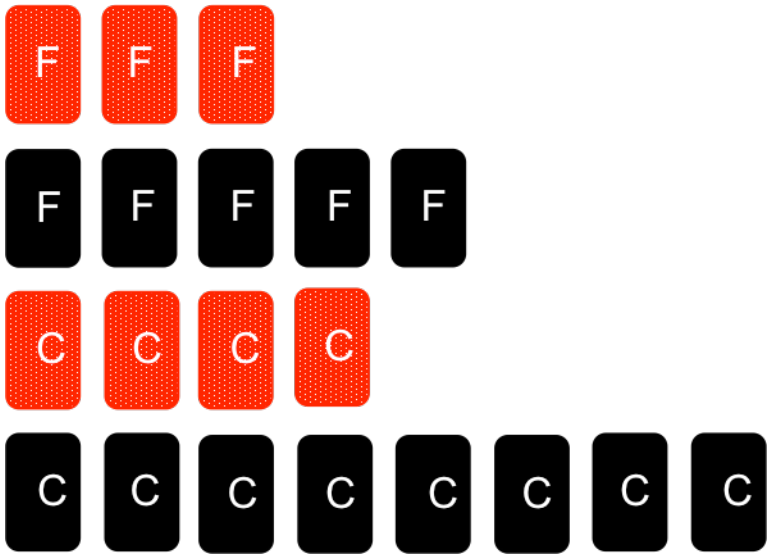
$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

where the probability of event $A$ occurring, conditional on the fact that event $B$ has already occurred, is equal to the product of the probability that event $B$ will occur, conditional on the fact that event $A$ has already occurred, and the unconditional probability of event $A$ occurring, divided by the unconditional probability of event $B$ occurring.[14]

Consider as an example of Bayes' Theorem a parking lot that contains 20 cars that consist of three red Fords, five black Fords, four red Chevrolets, and eight black Chevrolets. Figure 1 depicts the 20 cars in the lot. Tabs marked with an "F" denote Fords, and tabs marked with a "C" denote Chevrolets. The lightly dotted, red tabs denote red cars, and the solid black tabs denote black cars.

[13] *See, e.g.*, David R. Anderson, Dennis J. Sweeney & Thomas A. Williams, Statistics for Business and Economics 178–83 (South-Western 11th ed. 2011).
[14] *Id.*

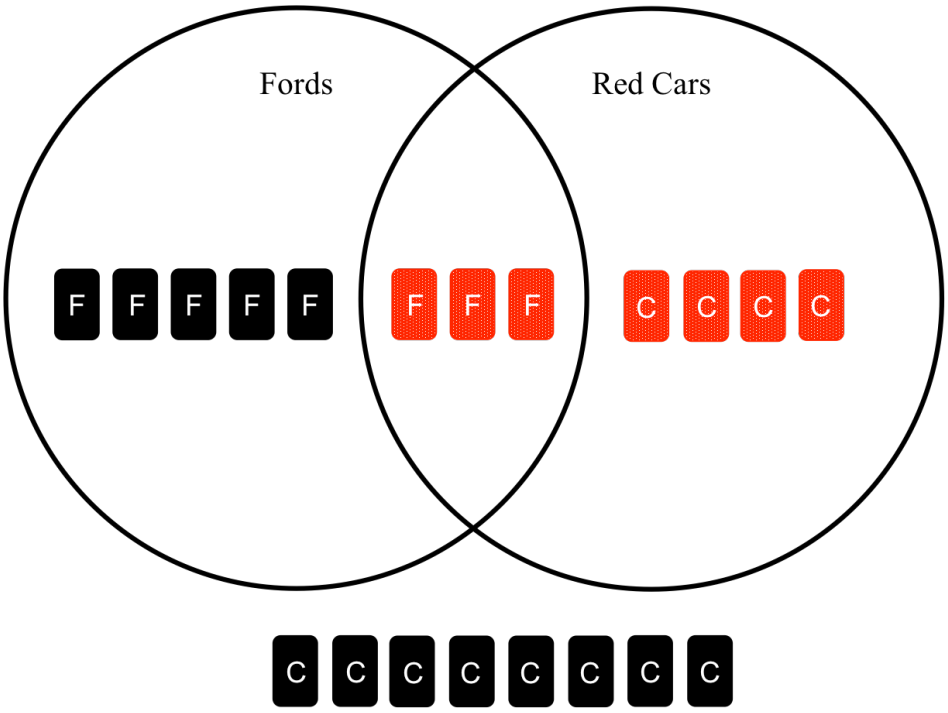
Figure 1. 20 Cars in the Parking Lot




The probability that a randomly chosen car is red is 0.35, the probability that a randomly chosen car is black is 0.65, the probability that a randomly chosen car is a Ford is 0.4, and the probability that a randomly chosen car is a Chevrolet is 0.6. Those probabilities are all unconditional probabilities determined by the relative prevalence of a single characteristic in the entire population. If a customer is given a random key to one of the 20 cars on the lot, and if all the keys look alike, he will know that there is a 35-percent chance that the key opens a red car.

However, suppose that the customer receives additional information about the manufacturer of the car. Now he can apply Bayes' Theorem to determine an updated, conditional probability that the key opens a red car. Assume now that the keys are no longer identical in appearance, such that the customer now can clearly identify the presence or absence of a Ford emblem on the key. He can therefore calculate the probability that the car is red with increased precision. Applying Bayes' Theorem, the probability that the car is red, given that it is a Ford, is equal to the product of the probability that the car is a Ford given that it is red (3 / (3 + 4) = 0.43) and the unconditional probability that the car is red (0.35), divided by the probability that the car is a Ford (0.4). Figure 2 shows a Venn diagram that identifies the cars that meet both or one of the two conditions—(1) that the car is a Ford and (2) that the car is red.

Figure 2. Venn Diagram of the Cars in the Lot,
on the Basis of Their Make and Color




The probability that a car is red given that it is a Ford is equal to 0.375, or 3 (the number of red Fords) divided by 8 (the number of total Fords), which significantly exceeds the probability of randomly selecting a red Ford without the additional information (namely, 0.15). The process of Bayesian updating thus enables more precise probability calculations. Similarly, it enables the parties to a patent license to incorporate more precise calculations of the probability of validity and infringement as new information becomes available.

### *B. Bayesian Updating of Patent Validity and Infringement*

Under the Leahy-Smith America Invents Act,[15] anyone may petition the PTAB to review one or more claims of a patent through the *inter partes* review process.[16] The IPR process might (1) affirm the validity of a patent's claims, (2) narrow the claims, or (3) invalidate the claims.[17] *Inter partes* review is often instituted in anticipation of, or in response to, a patent-infringement

[15] Pub. L. No. 112–29, 125 Stat. 284 (2011).
[16] 35 U.S.C. § 311.
[17] *See id.* § 318.

suit. In an IPR challenge, both the petitioner and the patent owner may present evidence to support their arguments concerning validity.[18] However, the petitioner need not be an accused infringer in a patent-infringement case pending in federal district court.[19]

An *inter partes* review that declines to invalidate particular claims of a given patent does not guarantee that the finder of fact in a patent-infringement suit in federal district court will find the same claims of that patent to be valid. However, a patent that survives IPR is more likely to be found valid at trial because at least one finder of fact, the PTAB, has rejected arguments that the challenged claims of the patent are invalid. Consequently, a patent that has survived an IPR will have a higher conditional probability of surviving the infringer's invalidity defense at trial in district court, and market participants will update their expectations of the patent's value accordingly.

The market's expectation of whether a patent is valid will become more accurate through the process of Bayesian updating, as new pieces of information on the expected value of the patent emerge from either the licensor's conclusion of voluntary arm's-length licenses with third parties or the rendering of validity decisions by the PTAB, by the courts, or by the U.S. International Trade Commission (ITC) or its administrative law judges. Because patent-infringement damages are calculated at trial under the assumption that the patent in suit is absolutely valid and infringed, there is at that time no remaining uncertainty regarding the patent's validity and infringement. If, before trial, the same patent survives an IPR, that piece of information makes the licensor and the third-party licensees marginally more certain that the patent ultimately will survive the infringer's invalidity defense at trial; consequently, after the IPR ruling but before trial, the patent holder will be able to execute arm's-length licensing agreements with third parties on the basis of an expected probability of validity that increasingly resembles the absolute validity that the *Georgia-Pacific* hypothetical negotiation explicitly assumes.

Similarly, as courts issue *Markman* rulings on claim construction for the patent in suit,[20] subsequent licensing agreements will revise the probability of infringement so that it more closely resembles the certainty of infringement that the *Georgia-Pacific* hypothetical negotiation also imposes by explicit assumption. If courts have narrowly construed the claims for the patent in suit, licenses executed after those *Markman* ruling might impute a low probability of infringement. Therefore, if actual infringement is found at trial, the observed royalty rate for licenses executed between the various

[18] *See id.* § 316.

[19] *See* J. Gregory Sidak & Jeremy O. Skog, *Attack of the Shorting Bass: Does the* Inter Partes *Review Process Enable Petitioners to Earn Abnormal Returns?*, 63 UCLA L. Rev. Disc. 120 (2015).

[20] Markman v. Westview Instruments, Inc., 517 U.S. 370 (1996).

*Markman* rulings and the finding of infringement at trial might significantly understate the patent's value in a hypothetical negotiation on the eve of first infringement that assumes validity and infringement with certainty.

Although sometimes only a short time elapses between claim construction and a trial for infringement, in more protracted litigation an *inter partes* review might occur far enough before an infringement trial that an economist can observe how the market's valuation of the patent in suit (in new licenses negotiated at arm's length with willing third parties) incorporates the updated conditional probability of that patent's infringement and validity. Applying statistical analysis to the changes in the market's valuation of the patent in suit over time will enable a rigorous examination of the royalty for that patent upon which a willing licensor and a willing licensee would agree, on the eve of first infringement, under *Georgia-Pacific*'s assumption of absolute validity and infringement.

## III. The Information That Observed Patent Licenses Reveal

Under factor 1 of the *Georgia-Pacific* framework, the licensor's royalty rates in licenses with third parties for the patent in suit are relevant to determining reasonable-royalty damages.[21] The *Georgia-Pacific* framework attempts to identify a reasonable royalty upon which the parties to the litigation would have willingly agreed to license the patent in suit in a hypothetical negotiation on the eve of first infringement.[22] That analysis of the hypothetical negotiation occurs during the damages portion of a patent-infringement trial and assumes with certainty that the patent in suit (or at least the relevant subset of the patent's claims being litigated) is valid and infringed. Consequently, a different set of factual assumptions underlies the hypothetical negotiation than exists in a real-world license between the licensor and a willing third party negotiating at arm's length.

In the real world, a patent has probabilistic value,[23] which means that the value paid for a license will incorporate the licensee's and the licensor's expectations that the patent is valid and infringed by some current or future product produced, used, or sold by the licensee. One can express a patent's expected value by multiplying (1) its value, conditional on its being valid and infringed, by (2) the probability that the patent is indeed valid and infringed. As uncertainty about the patent's validity and infringement decreases, that expected value will approach the patent's value under *Georgia-Pacific*'s

[21] Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970), *mod. and aff'd*, 446 F.2d 295 (2d Cir. 1971).

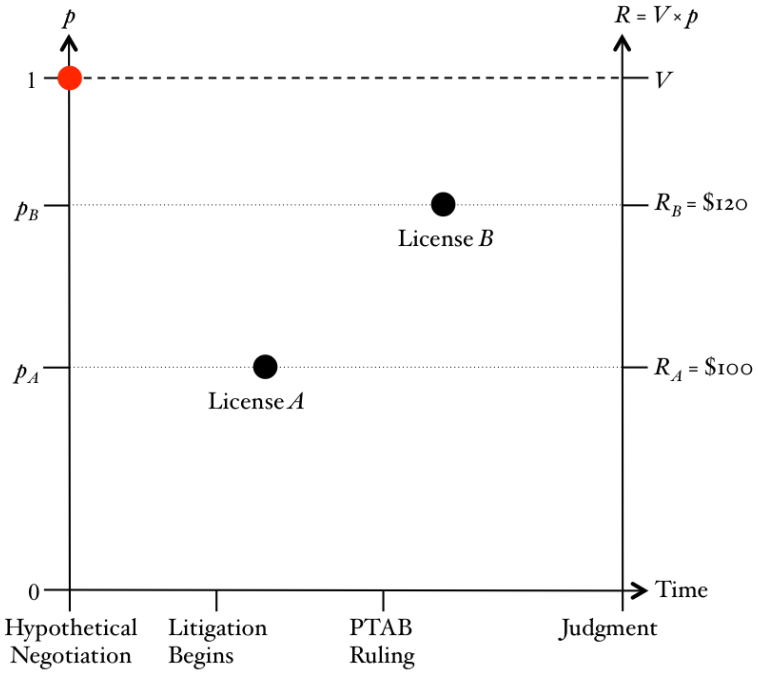
[22] For a general analysis of the hypothetical-negotiation framework for calculating damages for past infringement, see Sidak, *Bargaining Power and Patent Damages*, *supra* note 7.

[23] *See, e.g.*, Mark A. Lemley & Carl Shapiro, *Probabilistic Patents*, 19 J. Econ. Persp. 75 (2005).

assumption that the parties conduct the hypothetical negotiation knowing with absolute certainty that the patent in suit is valid and infringed. An observed royalty is then $R = V \times p$, where $R$ is the royalty, $V$ is the value of reasonable-royalty damages for the patent in suit given that the patent holder and infringer both know with absolute certainty that the patent is valid and infringed, and $p$ is the probability (at the time that the licensor and a given third party execute the license) that the finder of fact at trial will find the patent valid and infringed. For example, when the PTAB evaluates a patent through the IPR process, $p$ will increase if the patent is upheld or if review is denied. If the PTAB instead declares the patent to be invalid, then $p = 0$, and the patent with certainty no longer represents a legally protected property right. In that case, the "royalty," in effect, falls to zero. (The same analysis would apply to the district court's claim construction in its *Markman* order.)

Each new license for a particular patent reveals (1) how market participants value the technologies incorporated into the patent and (2) how market participants incorporate into their valuations of a patent the information revealed by events that occur during the interval since the most recent license agreement for that same patent. Figure 3 illustrates a stylized example of how one might observe changes in $p$ and the resulting changes in $R$ as the market incorporates over time new information concerning the likelihood that a given patent is valid and infringed.

Figure 3. Effect of New Information on the Observed Royalty and the Probability of a Patent Being Valid and Infringed




Suppose that a patent holder issues License *A* to a willing third party after commencing patent-infringement litigation against the infringer. Suppose that License *A* has a running royalty of $100 per unit. After the execution of License *A*, a company files an IPR petition requesting that the PTAB invalidate certain claims of the patent covered by License *A*. Suppose that the PTAB upholds the validity of those challenged claims. Later, the patent holder issues License *B* for the same patent to a different willing third party, but at a running royalty of $120 per unit. Suppose for simplicity that *V*—the commercial value of the patented technology—remains constant over time. Holding *V* constant, one can attribute the difference in the royalty rates between License *A* and License *B* entirely to the increase in the probability that the patent is valid (that is, the increase in *p*). In this case, the expected value $R = V \times p$ has increased from $100 to $120. If *V* remained constant over the time period between the two license agreements, one can infer that the market's estimate of the probability that the patent is valid and infringed has increased by 20 percent.[24]

[24] Let $V \times p_A = 100$ and $V \times p_B = 120$, where $p_A$ and $p_B$ represent $p$ at the time of each license. Then $V = 100/p_A$ and $V = 120/p_B$. Then $p_B/p_A = 120/100$.

Two points bear emphasis. First, $p$ represents the probability that the patent holder maintains a legally protected property right over the invention in question. Consequently, a failed IPR challenge, even if the PTAB rejected it on purely procedural grounds, will increase on the margin the market's estimate of the probability that a lawful right exists for the patent holder to receive a remedy under the Patent Act for infringement.

Second, because a patent has a finite term, each day that passes without a successful challenge to the patent's validity will increase the probability that the patent will be valid throughout its remaining life. Consequently, $p$ will generally increase over time, even if IPR challenges and claim constructions do not create exogenous shocks to market expectations concerning the patent's validity and scope. Because challenging a patent is costly, as the patent nears its expiration, an unlicensed user of the patent will have a lesser incentive to expend the resources necessary to challenge the patent since the benefit of doing so diminishes. This effect will also cause $p$ to increase over time. For a party accused of past infringement, the incentive to challenge the patent might not fall, but it will never increase as the patent approaches its expiration. It is a straightforward mathematical principle that the sum of a decreasing function and a nonincreasing function is a decreasing function. Therefore, it is intuitive that $p$ increases as the patent approaches its expiration.

There is some controversy regarding how much information about future events the finder of fact should assume to be available to the parties in the hypothetical negotiation occurring on the eve of first infringement. I have argued that, unless information about events that postdate the hypothetical negotiation informs what the parties expected *at the time of the hypothetical negotiation*, the so-called "Book of Wisdom" should be closed for the hypothetical negotiation.[25] Indiscriminately opening the Book of Wisdom might generate inefficient royalty rates that do not adequately compensate the patent holder.[26]

In sum, rulings on patent validity reveal information about the value of $p$ at the time of licensing. In the hypothetical negotiation, $p$ is assumed to equal 1. Therefore, as $p$ approaches 1 over time, the royalties negotiated in observed licenses for the patent in suit with similarly situated third-party licensees will more closely resemble a reasonable royalty emerging from the license contemplated in the hypothetical negotiation. Because a finding of invalidity will essentially drive $p$ to 0 for the patent claim in question, it is likely that, in any ongoing litigation, $p$ will increase as time passes. Assessing the events that occur—and how they affect $p$—will be a fact-specific inquiry.

[25] *See* J. Gregory Sidak, *How Relevant Is Justice Cardozo's "Book of Wisdom" to Patent Damages?*, 17 Colum. Sci. & Tech. L. Rev. 246 (2016).

[26] *Id.* at 284–90.

## IV. Calculating the Line of Best Fit

When different licensing agreements for a particular patent are observed over time, the expert economic witness can calculate the relationship between the royalty rates specified in those licenses and the revelation of new information indicating the probability that the patent is a legally protected property right. One can calculate the line of best fit with an ordinary least squares (OLS) regression using a limited number of observed licenses.[27] Calculating a line of best fit on the basis of observed licenses is typically more useful than calculating an average royalty specified in those licenses. Calculating an average assumes that the royalties do not change over time; in contrast, calculating a line of best fit enables the expert to identify whether the royalties for a given patent increase or decrease over time. Even if one calculates multiple averages—say, before and after a PTAB ruling on claims contained in the patent in suit—those average royalties miss that fact that the probability of the patent's validity and infringement will increase over time, even absent a court or PTAB ruling concerning the patent's validity or scope. Only an econometric model that includes the relationship between time and the observed royalties for the patent in suit will capture this effect.

In the simple case, the expert can identify a linear time trend through a bivariate regression. The OLS regression will identify the slope and intercept of the line that best fits the data, by minimizing the sum of the squared difference between the line and the actual data points.[28] The slope will represent how the conditional average royalty rate changes over time—that is, how the royalty is expected to change, on average, given the passage of time. In this case, each day that passes indicates an increase or decrease in the probability that the patent is valid and infringed. As the royalty rate increases or decreases over time with the addition of another voluntarily negotiated license for the patent in suit, the slope coefficient in the regression will indicate how much the market values the increasing probability that the patent is valid and infringed.

As I explained above, the increase in $p$ might result from decisions by a district court judge or by some other governmental body, such as the PTAB (or, for that matter, the ITC or one of its administrative law judges).[29] Moreover, even if no new decisions occur, the mere passage of time

[27] *See, e.g.*, Stock & Watson, *supra* note 17, at 114–15.

[28] *Id.*

[29] The ITC's findings do not bind a district court as a matter of res judicata or collateral estoppel in a parallel lawsuit, which typically proceeds more slowly than the ITC's investigation. *See* Texas Instruments Inc. v. Cypress Semiconductor Corp., 90 F.3d 1558, 1569 (Fed. Cir. 1996) (reaffirming "the rule that decisions of the ITC involving patent issues have no preclusive effect in other forums"). However, that legal rule does not prevent the possibility—as an economic matter—that the ITC's findings could reveal new information that would cause market participants to update their Bayesian estimates of the probability that the patent in suit is valid and infringed.

will correspond to a decreasing probability of a successful attempt to invalidate a patent. Therefore, even if no specific rulings are observed, the expert economic witness (and the finder of fact) should expect to observe an increase in the royalty rate as time passes, even after adjusting for inflation, as long as the value of the patent, $V$, remains the same.

Near the end of the patent's life, $V$ might not remain the same. Depending on the circumstances, waiting for a patent to expire is a kind of noninfringing substitute. As a patent nears its expiration, the value of the infringer's outside option—waiting for the patent to expire—will increase. The cost of the delay from waiting for the patent to expire will fall as the patent approaches expiration. Consequently, the infringer's maximum willingness to pay will decrease. This change will compress the bargaining range and, all other things being equal, decrease the patent's expected value, $pV$. However, over shorter intervals of time when the patent is not near expiration, the marginal effect on $p$ of any validity decisions should dominate the marginal effect on $V$ of waiting for the patent to expire.[30]

Figure 4 shows a stylized example of fitting a line to observed royalties. The $y$-axis measures per-unit royalties, and the $x$-axis measures the number of years since the defendant first infringed the patent in suit. The line of best fit allows the expert economic witness (and the finder of fact) to predict a royalty on any date by identifying the corresponding point on the line. One could extend the line of best fit into the future or the past to show how the market-disciplined belief concerning the patent's validity and scope has evolved over time with the revelation of new information. When enough observed licenses are available, the expert economic witness might be able to separate the effects of time from the effects of specific rulings on patent validity or scope. The time trend might generate an effect that is distinct from specific legal rulings. A particular ruling might shift the line of best fit upward or downward, and it might steepen or flatten the time trend.

[30] That is, $\partial R/\partial t = (\partial p/\partial t)V + (\partial V/\partial t)p > 0$, where $t$ is time. The marginal effect on the probability of validity and infringement will exceed the marginal effect on the value of the patent assuming absolute validity and infringement if $V/p > -\partial V/\partial t \div \partial p/\partial t$.

Figure 4. Lines of Best Fit Before and After a Shock Generated by the PTAB's Denying Institution of an IPR or Its Ruling That a Patent Is Not Invalid

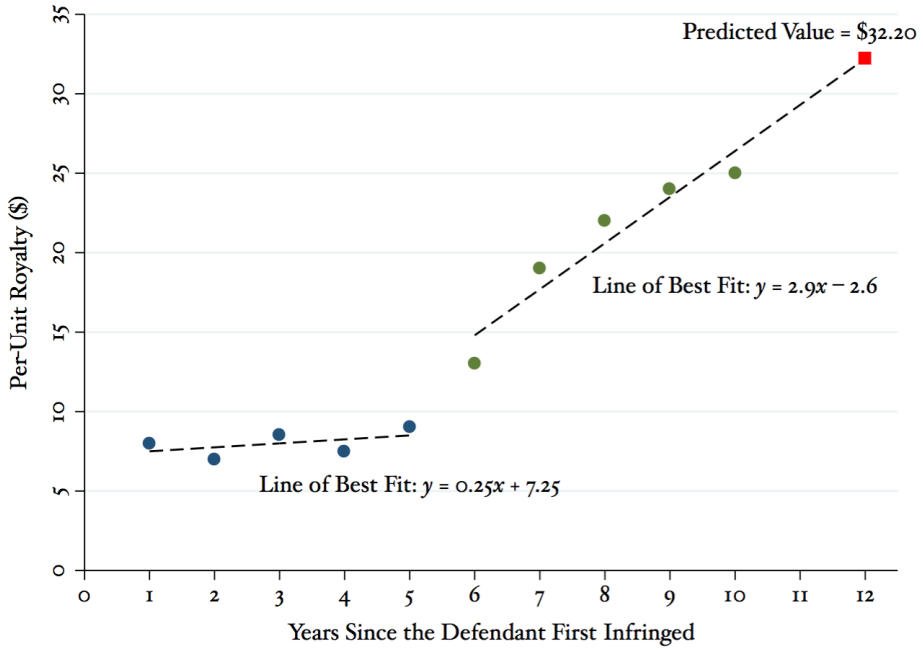


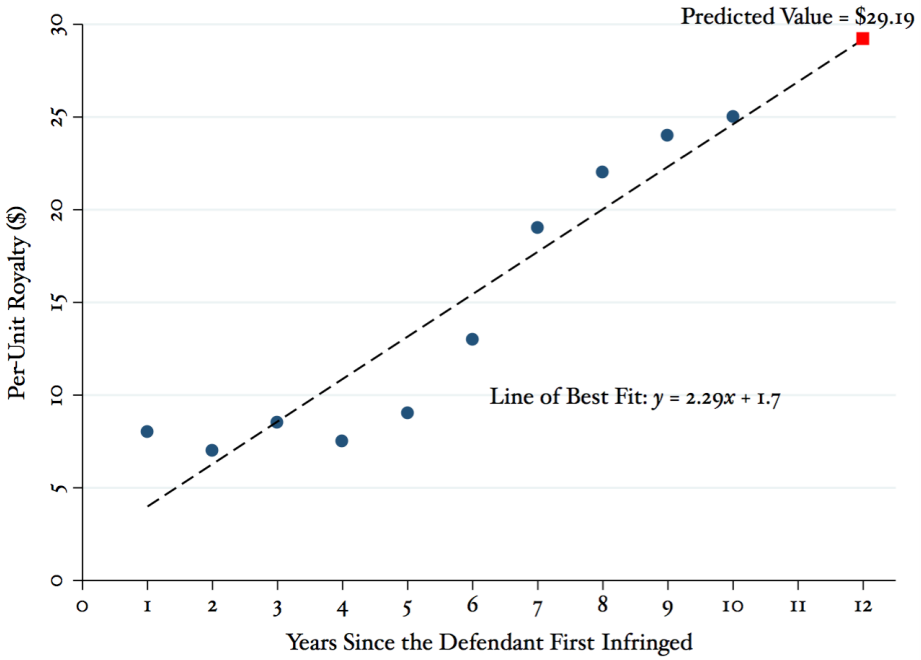

Figure 4 provides an example that shows not only how the passage of time increases the expected licensed royalty, but also how a hypothetical PTAB ruling (approximately 5.5 years after the date of the defendant's first infringement of the patent in suit) shifts the line of best fit upward. The line slopes upward both before and after the ruling and, in this example, becomes steeper after the PTAB's ruling. Calculating such a line requires a sufficient number of observations to identify a slope before and after the ruling, and an effect of the ruling itself. A single ruling will require the calculation of at least three slope coefficients. A slope coefficient for the passage of time will indicate how royalties changed with time before the ruling. A slope coefficient indicating whether the PTAB ruling in question has occurred will indicate by how much the line shifts as a result of the ruling. (The dummy variable takes the value of 1 if the PTAB ruling has occurred and a value of 0 if the ruling has not occurred.) Finally, a slope coefficient for the product of the variable indicating whether the PTAB ruling has occurred and the time trend, often called an interaction variable,[31] will identify the change in the slope that occurs as a result of the ruling.

[31] *See, e.g.*, Peter Kennedy, A Guide to Econometrics 252–53 (MIT Press 5th ed. 2003).

If there are enough observations after the PTAB's ruling, the data might indicate that a new pattern in the licensing of the patent in suit to willing third parties emerged after the PTAB's ruling on validity. The expert economic witness could then use only the observations that postdate the PTAB's ruling to predict what the per-unit royalty for the patent in suit will be on a particular date—most notably, on the date of trial in the patent-infringement litigation. In practice, if multiple rulings concern the patent's validity—a distinct possibility if that patent is the subject of current litigation—then one would need a large number of licenses to calculate separate slopes during each interval between rulings. Typically, however, one is unlikely to observe a large enough number of licenses over the short time period before trial to calculate a number of separate slopes. Consequently, the expert might be limited to calculating a single linear time trend.

When calculating a linear time trend, it might be the case that, although the data are best represented by a discontinuous function, the expert is constrained (due to limited degrees of freedom for a small number of observations) to calculate a continuous linear time trend. When such facts are present, the expert must rely on his training to identify and correct for any potential biases. Figure 5 contains the same observed licenses as Figure 4. However, the discontinuous line of best fit in Figure 4 is replaced with a continuous linear line of best fit in Figure 5. Determining which model (the continuous function or the discontinuous function consisting of two or more linear segments) is the proper model will depend on the facts of the case, including the availability of the necessary data. Although I use a standard linear OLS regression as a stylized illustration of how one can incorporate the timing of licenses and the role of new information into an economic analysis, one could use more advanced econometric functions if the situation warrants and the data are available. It is possible that the function that best fits the data is nonlinear. For example, one might fit the data in Figure 4 to an *S*-shaped sigmoid function.

Figure 5. Line of Best Fit for All Observations, as a Continuous Linear Function




Finally, it is important not to misapply common techniques for statistical inference when analyzing causal relationships (such as *t*-statistics) and techniques for analyzing the validity of regression predictions. Less commonly used measures of model precision, such as the root mean square error (RMSE), might measure the accuracy of a particular model more reliably than a measure with which a court might be more familiar, such as $R^2$. For example, Daniel Rubinfeld explains in the *Reference Manual on Scientific Evidence*, published by the Federal Judicial Center, that, "[a]s a general rule, courts should be reluctant to rely solely on a statistic such as $R^2$ to choose one model over another. Alternative procedures and tests are available."[32] In some regression models, identifying a causal relationship is the underlying goal of the regression model. However, at other times the expert economic witness is trying to make a prediction as precisely as possible, often with the limited number of observations that the factual record offers. Different statistical analyses will require different statistical tools.

[32] Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, *in* Reference Manual on Scientific Evidence 303, 345 (Federal Judicial Center 3d ed. 2011).

## Conclusion

Patent licenses reveal information about how the market values a patented technology and how the market values new information concerning the probability of a patent's validity and infringement. One can use that information to determine the value of the patent in suit under the assumed conditions in the *Georgia-Pacific* hypothetical negotiation that the patent is absolutely valid and infringed. Using regression analysis, an expert economic witness can use the change in royalty rates that occurs after pretrial rulings (by district courts, by the PTAB, or by the ITC or its individual administrative law judges) to calculate the market value of the increasing probability that the patent in suit is valid and infringed, and to predict the outcome of the hypothetical negotiation on the eve of the defendant's first infringement of the patent in suit. The line of best fit might predict a gradually increasing royalty over time, as uncertainty about the patent's validity and scope decreases. If so, extending the line of best fit to the trial date would provide a conservative (lower-bound) calculation of a reasonable royalty under the assumptions of absolute validity and infringement that apply in *Georgia-Pacific*'s hypothetical negotiation. This methodology enables the calculation of a reasonable royalty for the patent in suit that incorporates both the underlying legal assumptions of the hypothetical-negotiation framework and the market-disciplined prices that one subsequently observes in actual patent licenses voluntarily negotiated at arm's length between the licensor and willing third parties.