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Converting Royalty Payment Structures for Patent Licenses

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The parties to a patent-licensing agreement may choose from a variety of royalty structures to determine the royalty payment that the licensee owes the patent holder for using its patents. Three common structures of a royalty payment are (1) an *ad valorem* royalty rate, (2) a per-unit royalty, and (3) a lump-sum royalty. A royalty payment for a license might use a single royalty structure or a combination of these three structures.

Converting a royalty payment with one structure into an equivalent payment with another structure enables one to compare royalty payments across different licensing agreements. For example, in patent-infringement litigation, an economic expert can estimate damages for the patent in suit by examining royalties of comparable licenses—that is, licenses that cover a similar technology and are executed under circumstances that are sufficiently comparable to those of the hypothetical license in question.[1] However, licenses for a single patented technology might specify the royalty payment using different structures. One license might specify a per-unit royalty, a second might specify a lump-sum royalty, and a third might combine a lump-sum payment with a royalty rate. To analyze and compare the different royalty payments of those licenses, an economic expert or court must convert the royalties to a common structure. For example, a question related to the conversion of the royalty structure arose in August 2016 in *Trustees of Boston University v. Everlight Electronics Co.*, where, in granting an interlocutory appeal, the court asked "whether a district court can correct a damages figure on a motion for remittitur by extrapolating a royalty rate and base

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[1] *See, e.g.*, LaserDynamics, Inc. v. Quanta Comput., Inc., 694 F.3d 51, 79 (Fed. Cir. 2012).

from the jury's lump-sum award without express expert testimony explaining how to do so."[2]

Some courts have been skeptical that one can convert royalties across different structures. For example, also in August 2016 in *Baltimore Aircoil Co. v. SPX Cooling Technologies Inc.*, Judge Catherine Blake of the U.S. District Court for the District of Maryland excluded, in an order ruling on the patent holder's *Daubert* motion,[3] the opinion of the alleged infringer's economic expert, Kimberly Schenk of Charles River Associates, for using "lump sum agreements in calculating running royalty rates."[4] Judge Blake faulted Ms. Schenk for providing no justification for using the alleged infringer's sales projections in converting between the two royalty structures and concluded that her opinion "offer[ed] mere speculation masquerading as quantitative analysis."[5]

In this article, I explain how economic methodologies can enable an expert or a court to convert royalty payments reliably across different royalty structures. I show that such conversion of royalty payments requires not an accounting framework, but rather an *economic* framework. Projecting future sales, product prices, and market conditions are vital not only to producing accurate estimates of expected royalty payments, but also to converting those royalty payments across licenses that might specify different royalty payment schedules. Although those projection methods require additional judgment beyond a simple and straightforward calculation, converting royalty payments across different structures is a standard exercise that involves processes similar to those used to value patents outside adversarial proceedings.[6] The conversion of royalty payments across different structures need not be unreliable or inherently speculative.

In Part I, I describe three common structures of royalty payments for patents and analyze their main differences. In Part II, I explain how one can deconstruct a royalty payment into an equivalent payment with a different royalty structure in both simple and complex one-way licenses. In Part III, I show how to extend this framework to include the value of a cross license flowing back to the net licensor. I show that such methods enable courts to convert and reliably compare the royalty payments of different structures.

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[2] Nos. 12-11935, 12-12326, 2016 WL 4238554, at *2 (D. Mass. Aug. 9, 2016).

[3] Daubert v. Merrell Dow Pharms., Inc., 509 U.S. 579, 589–97 (1993) (establishing the district court as "gatekeeper" for admitting scientific expert testimony); *see also* J. Gregory Sidak, *Court-Appointed Neutral Economic Experts*, 9 J. Competition L. & Econ. 359, 384–86 (2013) (analyzing *Daubert* and its progeny).

[4] No. CCB-13-2053, 2016 WL 4426681, at *25 (D. Md. Aug. 22, 2016).

[5] *Id.*

[6] *See, e.g.*, Tim Heberden, *Intellectual Property Valuation and Royalty Determination*, *in* International Licensing and Technology Transfer: Practice and the Law ch. 4 (Adam Liberman, Peter Chrocziel & Russell Levine eds., Wolters Kluwer 2011).

I. Three Primary Structures of Royalty Payments

A patent license typically contains one or more of the following three royalty structures: (1) an *ad valorem* royalty rate, (2) a per-unit royalty, or (3) a lump-sum royalty. If the license specifies a royalty rate, the parties calculate the royalty payment as a percentage of a royalty base, which is typically the sales price of each sold product that practices the licensed technology. The patent holder charges the licensee that royalty payment in increments at a predetermined frequency, often on a yearly or quarterly basis. Under a royalty-rate structure, the royalty payment is positively correlated with both the price and the number of sold units of the product that practices the licensed patent. An increase in the quantity of units sold, an increase in the per-unit price of the patent-practicing product, or some combination the two will increase the total royalty payment. However, the licensee will not pay the patent holder any royalty if it does not sell any patent-practicing products.

When a license specifies a per-unit royalty, the royalty payment is dependent on and positively correlated with the number of shipped units—that is, the volume of patent-practicing products that the licensee sells during the term of the license agreement. Thus, the royalty payment that a licensee pays under the terms of a per-unit royalty, like that of a royalty rate, varies directly with the licensee's use of the patented technology. When the licensee's shipment volume increases or decreases, the total royalty that the licensee pays changes accordingly. However, unlike a royalty rate, a per-unit royalty is independent of changes in the sales price of the patent-practicing product.

In contrast to a royalty rate or per-unit royalty, a lump-sum royalty specifies a fixed, aggregate amount that the licensee must pay to obtain the right to use the patented technology during the term of the license. A lump-sum royalty removes the administrative burden and costs of monitoring the actual use of the licensed technology because the royalty payment is independent of the licensee's *actual* sales. The licensing parties typically agree upon the amount of the lump-sum royalty before the royalty-bearing sales occur—that is, they typically calculate a lump-sum payment in advance by using the licensee's *projected* sales revenue or unit shipments for the duration of the license.[7] The licensee typically makes that payment at the beginning of the license term or according to a predetermined payment schedule. The licensee will pay the full amount of the predetermined lump-sum royalty regardless of the extent to which it actually uses the licensed technology.

[7] *See, e.g.*, Linkco, Inc. v. Fujitsu Ltd., 232 F. Supp. 2d 182, 188 (S.D.N.Y. 2002) ("[A] reasonable royalty may be computed in various ways, including a lump-sum royalty based on expected sales.").

Thus, a lump-sum royalty might not reflect accurately the licensee's *ex post* use of the patented technology.[8]

II. Converting Royalty Payments of a One-Way License

Using economic methodologies, one can convert a royalty with any given structure into an equivalent royalty that uses a different structure. For example, one can convert a royalty payment that is specified as a per-unit royalty into an equivalent royalty payment under a different structure, such as an *ad valorem* royalty rate. I will use the term *derived royalty* to indicate a royalty that one obtains from the deconstruction or transformation of a royalty payment. Because the derived royalty and the original royalty payment of a license imply the same expected payment at the time of a license's issuance, the parties to a patent-licensing agreement will be indifferent between the two royalty payments.

I begin my analysis by examining a one-way license—that is, a license in which the parties determine the royalty that the licensee will pay the patent holder to use its licensed patents. The parties might determine the royalty payment using a single royalty structure or by using a complex structure that combines multiple royalty structures.

A. Licenses That Use a Single Royalty Structure

Simple economic methodologies enable the conversion of royalties in one-way licenses that use a single royalty structure. Suppose that a license specifies a per-unit royalty and that one must convert that royalty into an equivalent *ad valorem* royalty rate. To do so, one should compare the expected royalty payments under the two royalty structures and find the royalty rate that makes the two payments equal under appropriate assumptions. For example, when the license specifies a per-unit royalty, the expected royalty payment that the patent holder will receive equals the per-unit royalty multiplied by the projected number of the patent-practicing product's sold units, which the parties estimate at the time of the license's issuance. Equation (1) states this relationship:

$$\textit{Per-Unit Royalty Fee} \times \textit{Projected Number of Units} = \textit{Expected Royalty Payment}. \quad (1)$$

Conversely, when the license specifies an *ad valorem* royalty rate, the expected royalty payment equals the projected price of the licensed product multiplied

[8] *See* J. Gregory Sidak, *How Relevant Is Justice Cardozo's "Book of Wisdom" to Patent Damages?*, 16 Columbia Sci. & Tech. L. Rev. 246 (2016).

by the projected number of sold units (for simplicity, I will call this algebraic product the licensee's projected sales revenue) and by the royalty rate, as Equation (2) shows:

$$\textit{Projected Revenue} \times \textit{Royalty Rate} = \textit{Expected Royalty Payment}. \quad (2)$$

Setting Equations (1) and (2) equal, one can derive the following relationship:

$$\textit{Per-Unit Royalty Fee} \times \textit{Projected Number of Units} = \textit{Projected Revenue} \times \textit{Royalty Rate}. \quad (3)$$

Therefore, one can derive an *ad valorem* royalty rate simply by dividing the total projected royalty payment by the projected revenue. Equation (4) expresses that relationship:

$$\frac{\textit{Per-Unit Royalty Fee} \times \textit{Projected Number of Units}}{\textit{Projected Revenue}} = \textit{Derived Royalty Rate}. \quad (4)$$

Because the licensee's projected revenue equals the projected number of sold units of the patent-practicing product multiplied by the projected price per unit, one can state the relationship of Equation (4) more simply as:[9]

$$\frac{\textit{Per-Unit Royalty Fee}}{\textit{Projected Price Per Unit}} = \textit{Derived Royalty Rate}. \quad (5)$$

Thus, simply using the projected unit price of the licensed product enables one to convert a per-unit royalty fee into a derived royalty rate.

Similarly, one can deconstruct a lump-sum royalty payment into a derived royalty rate. A licensee might make a lump-sum payment either collectively at the beginning of the license's term or progressively following a schedule over that term. In either case, one can calculate the present value of projected revenues over the license's term using the discounted cash flow (DCF) method by applying an appropriate discount rate,[10] as Equation (6) shows:

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[9] The following equation illustrates the substitution and reduction process:

$$\frac{(\textit{Per-Unit Royalty Fee})\ \sout{(\textit{Projected Number of Units})}}{(\textit{Projected Price Per Unit})\ \sout{(\textit{Projected Number of Units})}} = \frac{\textit{Per-Unit Royalty Fee}}{\textit{Projected Price Per Unit}}$$

[10] *See* William Choi & Roy Weinstein, *An Analytical Solution to Reasonable Royalty Rate Calculations*, 41 J.L. & Tech. 49, 56 (2001) (emphasizing that a DCF method is used to "discount, into present value, the expected cash flow from a licensing agreement"); *see also* Heberden, *supra* note 6, at 21 ("[The discount rate] is a function of three factors: the risk free rate (yield on government bonds), the market risk premium (extra risk applying to the share market), and specific risks attached to the company and [(intellectual property)] IP.").

$$\sum_{\textit{First Payment}}^{\textit{Last Payment}} \frac{\textit{Total Projected Revenue}}{(1 + \textit{Discount Rate})^{\textit{Time}}} = \textit{Present Value of Projected Revenue.} \quad (6)$$

Dividing the lump-sum royalty payment by the present value of the licensee's projected sales revenue following Equation (7), one can calculate the derived royalty rate of a lump-sum royalty payment:

$$\frac{\textit{Lump-Sum Royalty Payment}}{\textit{Present Value of Projected Revenue}} = \textit{Derived Royalty Rate}. \quad (7)$$

Note that Equations (5) and (7)—which one can use to convert a per-unit royalty and a lump-sum royalty into a derived royalty rate—are easily invertible. For example, multiplying a royalty rate by the projected per-unit price gives a derived per-unit royalty of equivalent value at the time of the license's issuance. Similarly, multiplying a royalty rate by the present value of the licensee's projected revenues gives a derived lump-sum royalty of equivalent value. Finally, it is possible to convert a lump-sum royalty into an equivalent per-unit royalty—and vice versa—by using either Equation (5) or Equation (7) to convert that royalty payment into a derived royalty rate as an intermediate step.

In sum, economic projections enable one to convert seamlessly an observed royalty from any of the three main royalty structures into one under another structure.

B. Licenses That Use a Complex Royalty Structure

By employing similar techniques, an economic expert can likewise deconstruct a royalty payment for a complex license—that is, a license that combines multiple royalty structures to determine a royalty payment. For example, suppose that a license specifies both a lump-sum payment of $500,000 and an additional per-unit royalty of $1. One can deconstruct that license in two steps to calculate a derived royalty rate. First, one must convert each component of the complex royalty that corresponds to each structure that the complex royalty uses into an equivalent royalty rate using the techniques outlined above in Part I.A. One can then calculate a derived royalty rate by simply summing the calculated royalty rates of each component. For example, if an economic expert finds that the lump-sum payment of $500,000 is equivalent to a royalty rate of 3 percent, and the per-unit royalty of $1 is equivalent to a royalty rate of 2 percent, then the derived royalty rate would be 5 percent for the license agreement.

A complex license might also combine an *ad valorem* royalty rate with a royalty cap—that is, an upper limit on the royalty fee that the licensee would pay for each licensed product. Under that structure, the royalty rate determines the per-unit royalty payment unless that payment exceeds the specified cap, in which case the royalty payment becomes a fixed, per-unit fee equivalent to the (binding) cap. One can estimate the value of a royalty defined under this structure by calculating a probability-weighted expected royalty fee—that is, by multiplying the royalty fee associated with each possible unit price by the respective probability that the product will sell at that price and then taking the sum of the resulting products.

For example, consider a technology that is licensed at a 1-percent royalty rate with a $2 cap on the per-unit royalty fee. Suppose that there is a 60-percent probability that the product incorporating this technology will sell at $180 per unit and a 40-percent chance that it will sell at $220 per unit. If the product sells at $180 per unit, then the licensee pays a $1.80 per-unit royalty fee ($180 × 1% = $1.80). Alternatively, if the product sells at $220 per unit, the royalty rate would yield a $2.20 per-unit royalty fee ($220 × 1% = $2.20), which exceeds the $2 cap. In that case, the royalty cap takes effect, and the licensee pays a flat $2.00 royalty fee, which is equivalent to the binding cap. Taking a probability-weighted average of those results, one can calculate an overall per-unit royalty fee of $1.88.[11]

Although deconstructing the royalty payment for a complex license can be burdensome, as it requires more data and estimation of the probabilistic price distribution, the theoretical basis is straightforward. If sufficient data are available, this methodology produces an economically sound value for the derived royalty rate.

III. Converting Royalty Payments of a Cross License

Unlike a one-way license, a cross license assigns to each party the right to use the counterparty's patents. Implicit in a cross license is the idea that each party pays to use the counterparty's licensed patents. However, the one-way royalties that the two parties pay each other are not determined separately in the license. Rather, the license specifies only a single balancing royalty—that is, the ultimate royalty that one party (the net payor) must pay to the counterparty (the net payee). Specifying only the balancing royalty payment reduces the license agreement's transaction costs and accounting costs by simultaneously accounting for both parties' royalty payments.

[11] That is, ($1.80 60%) + ($2.00 40%) = $1.88.

A. A Framework for Understanding the Balancing Royalty

The following analogy illustrates the concept of a balancing royalty.[12] Consider a driver who wants to replace his old BMW 328i with a new Toyota Camry. At the dealership, he decides to accept the dealer's offer to trade in his used car and receive a credit (a trade-in allowance) toward the price of the Camry. The dealer and the driver are each, in effect, buying and selling simultaneously in this transaction. The dealer offers to buy the used BMW at a price equal to the trade-in allowance. The better the condition of the used BMW, the higher the credit the dealer will grant the driver toward the net price—that is, the total amount of cash that the driver will pay for the new Camry. If the BMW's fenders were rusted, the dealer would offer less than if the car were in a pristine condition.

An analogous transaction occurs when two patent holders cross license their respective portfolios. Each patent portfolio commands a particular one-way royalty payment from the counterparty. Typically, the royalty that a cross license specifies is a balancing royalty that one party must pay to the other—that is, the difference between the two opposing one-way royalties that each party owes the other for the use of its respective patent portfolio. The value that each party's patent portfolio generates for the other party determines the net-paying and net-receiving parties, as well as the amount of the balancing royalty. As Figure 1 illustrates, the balancing royalty is analogous to the net price that the driver pays the dealer for the new Camry.

12 *See* J. Gregory Sidak, *How Licensing a Portfolio of Standard-Essential Patents Is Like Buying a Car*, WIPO, June 2015, at 11.

Figure 1. The Balancing Royalty Payment in a Cross License

Price of New Car	–	Value of Trade-In	=	Net Amount Due
	–		=	Payment
Value that Party *A*'s Patented Technology Generates for Party *B*'s Products	–	Value that Party *B*'s Patented Technology Generates for Party *A*'s Products	=	Balancing Royalty Payment

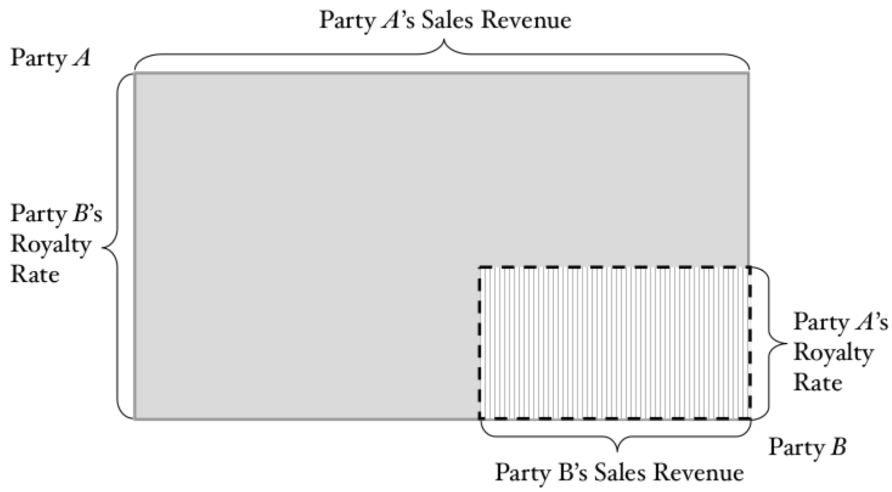
Sources: *BMW Pre-Owned Inventory*, BMW of Visalia, http://bmwofvisalia.com/Used-Cars/2014-BMW-328i-Visalia-MtnSLvfCnk_h4ZcN_gg4uA; *New 2016 Toyota Camry LE Sedan Front-Wheel Drive in Klamath Falls*, Lithia Toyota of Klamath Falls, https://www.lithiatoyotaklamathfalls.com/new/Toyota/2016-Toyota-Camry-klamath-falls-OR-7992e7a90a0e0adf-001605dee09e18b0.htm.

The following equation captures the relationship between the balancing royalty, royalty rates, and sales revenues of the two parties to the license:

$$\begin{aligned}(\textit{Royalty Rate of } \mathrm{B} \times \textit{Sales Revenue of } \mathrm{A}) - \\ (\textit{Royalty Rate of } \mathrm{A} \times \textit{Sales Revenue of } \mathrm{B})\end{aligned} = \textit{Balancing Royalty}. \qquad (8)$$

Figure 2 illustrates this relationship. Party *A*'s *one-way* royalty payment for the use of Party *B*'s patent portfolio is the area of the entire box in Figure 2. That is, the royalty equals Party *A*'s sales revenue from its licensed products multiplied by Party *B*'s royalty rate. Party *B*'s *one-way* royalty payment for the use of Party *A*'s portfolio is the area of the dotted box. Similarly, that amount is Party *B*'s sales revenue from its licensed products multiplied by Party *A*'s royalty rate. The difference in the areas of the two boxes, shaded in solid, equals the balancing royalty that Party *A* (the net payer) pays Party *B* (the net payee) under the cross license.

Figure 2. The Balancing Royalty in a Cross License



The magnitude and direction of the balancing royalty depend on the difference between the royalty payment that Party *A* owes Party *B* and the royalty payment that Party *B* owes Party *A*. Suppose that Party *B*'s patent portfolio has a higher royalty rate, and that Party *A* has higher sales revenues from its licensed product. Given those conditions, Party *A* gains higher value from the licensed patents and must therefore pay the balancing royalty to Party *B*, as Figure 2 shows.

The balancing royalty in the cross license is necessarily less than the *one-way* royalty that the net-receiving party would charge the net-paying party for using its licensed patents. It bears emphasis that the net-receiving party is determined on the basis of both (1) the relative strength of each party's patent portfolio and (2) the amount of each party's sales. Assume, for example, that Party *A* charges a royalty rate of 10 percent, while Party *B* charges a rate of 1 percent. Assume further that Party *A* generates sales revenues of $1,000 from its patent-practicing product and that Party *B* generates only $100. Applying Equation (8) shows that, although Party *A* possesses a stronger patent portfolio, the balancing royalty in this situation would be zero percent. Therefore, the net recipient is not necessarily the party with the strongest patent portfolio. In a cross license where the parties' sales are assumed to be equal, the closer the value of the weaker patent portfolio is to the value of the stronger patent portfolio, the lower the balancing royalty rate because the value of the exchanged technology accounts for a larger portion of the implicit royalty payment.

B. *Relative Patent-Portfolio Strength Between the Two Parties to a Cross License as the Second Dimension of a Lump-Sum Royalty*

Deconstructing a lump-sum royalty in a cross license into a derived one-way royalty rate requires an economic expert to consider a second factor that is absent in the case of one-way licenses—the relative patent-portfolio strength of the two parties to the cross license. Two variables remain unknown in the calculation of the balancing payment: (1) the implicit royalty rate that Party *A* sets and (2) the implicit royalty rate that Party *B* sets. Because the royalty rate at which a patent holder licenses its patent portfolio reflects the strength of that patent portfolio, the ratio of those two implicit rates is equal to the strength of Party *A*'s patent portfolio relative to Party *B*'s patent portfolio, as Equation (9) shows:

$$\frac{\textit{Royalty Rate of Party } \text{A}}{\textit{Royalty Rate of Party } \text{B}} = \textit{Relative Portfolio Strength}. \tag{9}$$

Assuming that the royalty a firm charges for licensing its portfolio is positively related to the strength of that portfolio, an economic expert can deconstruct the implicit royalty rates using a separate exogenous measure of the portfolios' relative strengths. Although the relative portfolio strength of two parties can change over time, those changes are likely to occur slowly due to the pace of patent filings and patent issuances.[13]

It also bears emphasis that, although the relative portfolio strength serves as a practical *proxy* for the ratio of the royalty rates, the correspondence is unlikely to be perfect. That is, the relative strength of a patent portfolio and the actual royalty rates might depend on other factors, such as the relative demand for the patented technology. For example, suppose that the two parties are smartphone manufacturers. One produces basic, low-end smartphones, whereas the other produces high-end smartphones. Suppose further that the patented technology in question is part of the 4G standard. Assuming that 4G readability forms a greater percentage of the value of low-end smartphones, a greater royalty rate might be justified for the low-end smartphone manufacturer.

For the purposes of my analysis, I treat relative portfolio strength as an exogenous parameter that the expert must estimate. One can rearrange Equation (9) such that the royalty rate that Party *A* sets is some multiple of the royalty rate of Party *B*:

[13] For ease of exposition, I assume here that the relative portfolio strength has a direct and linear relationship with respect to the ratio of the parties' one-way royalty rates. Intuitively, a firm with a stronger patent portfolio will be able to charge a higher royalty for a license to its patents. One could replace this direct and linear measure of relative portfolio strength with a nonlinear functional form if the necessary data were available to estimate that nonlinear functional form.

$$\textit{Royalty Rate of } \mathrm{A} \quad = \quad \textit{Relative Portfolio Strength} \times \textit{Royalty Rate of } \mathrm{B}. \qquad (10)$$

Using Equation (10), one can solve for the derived one-way royalty rates. For example, to solve for *Royalty Rate of* B, one can first substitute the term for *Royalty Rate of* A in Equation (8) into Equation (10), so that:

$$\textit{Balancing Royalty} \quad = \quad (r_B \times S_A) - (\textit{Relative Portfolio Strength} \times r_B \times S_B), \qquad (11)$$

where r_B denotes the royalty rate of Party *B*'s patent portfolio, and S_A and S_B denote the sales revenue of Party *A* and Party *B*, respectively. Assuming that relative portfolio strength is known, there is only one unknown variable in this equation: the royalty rate of Party *B*'s patent portfolio (r_B). One can thus solve for that royalty rate in terms of the other known variables. Rearranging to isolate the variable r_B, Equation (11) becomes:

$$\frac{\textit{Balancing Royalty}}{S_A - (\textit{Relative Portfolio Strength} \times S_B)} \quad = \quad r_B. \qquad (12)$$

Therefore, by making certain assumptions about the relationship between the portfolio strength of any two counterparties, one can transform a balancing royalty payment in a cross license into a derived one-way royalty rate.

IV. Conclusion

Basic economic techniques equip a damages expert with a reliable means of converting a royalty payment with one structure to a royalty of equivalent value under a different structure. I have shown that one can use reliable methodologies to convert royalty payments across structures both (1) in cases where the parties have executed a one-way license—that is, a license in which the parties determine the conditions for the licensee's use of the patent holder's patents—and (2) in cases where the parties have executed a cross license, which grants each party the right to use the counterparty's patents. When the license uses a simple royalty structure, converting the royalty payment across different structures is also relatively simple. For example, information about the patent-practicing product's projected price might suffice to convert a per-unit royalty fee to a derived royalty rate. However, when the license in question is a cross license or complex license, converting royalty payments to a derived royalty rate might be more burdensome, as it might require the estimation of additional parameters, such as relative portfolio strength. Nevertheless, with sufficient data, existing economic methodologies offer a reliable basis for estimating and comparing the derived royalty rates of different licenses. Courts can supplement the derived royalty rate with adjustments to account for a company's bargaining power or market risk implicit in the different royalty structures.

V. Appendix

Here, I present a concise set of equations to summarize each royalty-structure-conversion process. This appendix can serve as a reference for economic experts and courts when converting royalty payments across different structures.

Table A1. Variables Used in Calculations

Variable	Definition
R_u	Per-Unit Royalty Fee
R_l	Lump-Sum Royalty Payment
r_s	*Ad Valorem* Royalty Rate
r_d	Derived Royalty Rate
R_c	Per-Unit Royalty Cap
$E[u]$	Projected Number of Units
$E[R]$	Expected Royalty Payment
$E[S]$	Projected Revenue
$E[p]$	Projected Price Per Unit
$E[S_o]$	Present Value of Projected Revenue
d	Discount Rate
t	Time
w_i	Probability of Event i
r_A	Royalty Rate of Party A
r_B	Royalty Rate of Party B
S_A	Sales Revenue of Party A
S_B	Sales Revenue of Party B
R_b	Balancing Royalty Payment
x	Relative Portfolio Strength

A. *Converting Royalty Payments of One-Way Licenses*

1. *Deriving a Royalty Rate from a Per-Unit Royalty*

$$R_u \times E[u] = E[R] \tag{A1}$$

$$\frac{R_u \times E[u]}{E[S]} = \frac{E[R]}{E[S]} = r_d \tag{A2}$$

$$\frac{R_u \times E[u]}{E[p] \times E[u]} = \frac{R_u}{E[p]} = r_d \tag{A3}$$

2. *Deriving a Royalty Rate from a Lump-Sum Royalty*

$$\sum_{t=1}^{T} \frac{E[S]}{(1+d)^t} = E[S_o] \tag{A4}$$

$$\frac{R_l}{E[S_o]} = r_d \tag{A5}$$

3. *Converting Across the Three Royalty Structures*

$$\frac{R_u}{E[p]} \approx r_s \approx \frac{R_l}{E[S_o]} \tag{A6}$$

4. *Deriving a Royalty Rate of a Complex License with Multiple Structures*

$$R_{total} = R_{Per\text{-}Unit} + R_{Ad\text{-}Valorem} + R_{Lump\text{-}Sum} \tag{A7}$$

$$r_d = \frac{R_u}{E[p]} + r_s + \frac{R_l}{E[S_o]} \tag{A8}$$

$$r_d = r_{d,Per\text{-}Unit} + r_{d,Ad\text{-}Valorem} + r_{d,Lump\text{-}Sum} \tag{A9}$$

5. *Deriving a Per-Unit Royalty Fee of a Complex License with a Per-Unit Royalty Cap*

$$E[R_u] = \sum_i R_{u,i} w_i \quad (R_c \geq R_{u,i}) \tag{A10}$$

B. *Deriving a Royalty Rate of a Cross License*

$$\frac{r_A}{r_B} = x \tag{A11}$$

$$r_B S_A - r_A S_B = R_b \tag{A12}$$

$$r_B S_A - x\, r_B S_B = R_b \tag{A13}$$

$$\frac{R_b}{S_A - x S_B} = r_B \tag{A14}$$