Hedonic Prices and Patent Royalties

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A hedonic model explains a good's price in terms of its characteristics.[1] In this article, we use hedonic prices to estimate the permissible range for a reasonable royalty for a standard-essential patent (SEP) subject to its owner's commitment to offer to license the patent on reasonable and nondiscriminatory (RAND) terms. Our methodology is equally applicable to the calculation of fair, reasonable, and nondiscriminatory (FRAND) royalties for SEPs. The immediate purpose of our analysis is to determine whether, as a matter of contract law, a particular offer that the SEP holder has made has discharged its obligation to its standard-setting organization (SSO) to make an offer to license its SEPs on RAND or FRAND terms to a third party seeking to implement the standard. However, if asked or required to set a specific RAND or FRAND rate for a specific portfolio of SEPs, a court or arbitral panel could take our analysis one step further, by determining where within the RAND or FRAND bargaining range a bilaterally negotiated royalty between the parties would most likely fall.[2]

More generally, our methodology shows how one can use hedonic prices to calculate the permissible range for reasonable royalties for the infringement of patents that are *not* encumbered by a RAND or FRAND obligation,

even if no comparable licenses exist. In short, our methodology of using hedonic prices to calculate the permissible range for reasonable royalties for patent infringement has potentially broad applicability.

Our hedonic price estimation enables a finder of fact to separate the incremental value of the patented invention from the underlying value of standardization. In this respect, our hedonic price methodology provides a rigorous means to satisfy the Federal Circuit's directive in *Ericsson v. D-Link* to "consider the difference between the added value of the technological invention and the added value of that invention's standardization."[3] Our methodology also applies to measuring a reasonable royalty for a patent that is practiced in a multicomponent product but is not declared essential to any standard.

Our hedonic price methodology provides an empirically robust and reliable measurement of the incremental value of a patented technology for calculating a RAND or FRAND royalty. That is not to say that there are no other acceptable methodologies for doing so if the necessary information is available. For example, license agreements for a patented technology can provide accurate and reliable evidence of that technology's value if they are sufficiently comparable to the license at issue in suit.[4] Comparable licenses identify the price that implementers have demonstrated that they are willing to pay for the right to manufacture a standard-compliant product. However, comparable licenses might not exist—and by definition will not exist if the SEPs are being licensed for the first time. This situation is likely in the case of a nascent standard or a smaller company that lacks a history of licensing. In such a case, our hedonic price methodology provides an alternative methodology for determining the value of the patented inventions. Our hedonic methodology also can be used in situations separate from litigation, such as in valuation analysis to support licensing negotiations.

This article draws from analysis upon which one of us (Sidak) relied in expert economic testimony submitted in 2017 to the U.S. International Trade Commission (ITC) on behalf of the complainant (Netlist Inc., in *Certain Memory Modules and Components Thereof, and Products Containing Same*, Investigation No. 337-TA-1023) and was completed before the issuance of an initial decision. That investigation addressed the complaint that the respondent had infringed certain patents used in memory modules for enterprise servers. The complainant had declared the patents in suit to be essential to

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[3] Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1233 (Fed. Cir. 2014); *see also* J. Gregory Sidak, *The Value of a Standard Versus the Value of Standardization*, 68 Baylor L. Rev. 59 (2016).

[4] *See* J. Gregory Sidak, *Apportionment, FRAND Royalties, and Comparable Licenses After* Ericsson v. D-Link, 2016 U. Ill. L. Rev. 1809, 1821 ("The district court and the Federal Circuit opinions in *Ericsson v. D-Link* confirm that it is a reliable methodology to use comparable licenses to calculate a FRAND royalty.").

standards established by the Joint Electron Devices Engineering Council (JEDEC).

We use hedonic price estimation to identify the demonstrated value that customers place on various product features, including the standard for load-reduced dual-inline memory modules (LRDIMMs) using the fourth generation of dual data rate (DDR4) dynamic random-access memory (DRAM) chipsets. We isolate the value of the technologies that allow JEDEC's LRDIMM standard to operate, above and beyond the value of standardization itself (that is, the value of having *any* standard, regardless of its precise technological content) by determining the premium that customers are willing to pay, relative to the next-best standard, which in this case is JEDEC's earlier standard for registered dual-inline memory modules (RDIMMs). After netting out the manufacturer's incremental cost of producing a DDR4 LRDIMM rather than an RDIMM, we then use forward-citation analysis of patents essential to the LRDIMM standard to calculate the maximum possible royalty that would be RAND for the complainant's portfolio of SEPs.

We use this analysis to answer the positive question of what price customers are willing to pay to access a standard for the purpose of meeting, as a legal matter, the evidentiary requirements for identifying and apportioning value. We do not attempt to address the normative questions of how that value should be distributed in a particular negotiation, or whether patent owners or manufacturers should gain the value from standardization or from technology in the public domain.

Throughout this article, we explain our analysis but mask (through hypothetical numerical values) any discussion of actual royalty offers made in the negotiations preceding the ITC's 1023 investigation. By so doing, none of our discussion or economic analysis relies on confidential business information (CBI) from that investigation.

In Part I of this article, we describe the provisions of a SEP holder's RAND obligation to JEDEC and explain that those provisions neither guide nor constrain the proper economic methodology for calculating a RAND royalty. In Part II, we explain in nontechnical terms the theory of hedonic prices, which economist Sherwin Rosen greatly advanced in 1974.[5] In Part III, we explain how to estimate hedonic prices to determine the value that a patented invention creates. In Part IV, we illustrate how to use hedonic price analysis to measure the contributions of patented advances practiced in LRDIMMs. In Part V, we use a common research methodology found in the economic scholarship on patent valuation—forward-citation analysis of patents—to apportion the value that an SEP holder's portfolio contributes

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[5] Sherwin Rosen, *Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition*, 82 J. POL. ECON. 34 (1974).

to the standard in question. In Part VI, we use a bargaining-range framework to demonstrate that the complainant's hypothetical RAND offer of $12 per unit in the ITC's 1023 investigation comports with the incremental value attributable to the complainant's LRDIMM SEP portfolio. In Part VII, we explain why our hedonic price analysis methodology produces admissible expert testimony under the Federal Rules of Evidence, why that methodology also satisfies the Federal Circuit's apportionment requirement enunciated in *Ericsson v. D-Link*, and why an alternative methodology that some academics advocate—an analysis of a patent's *ex ante* incremental value at the time of the standard's adoption—is flawed on economic grounds and consequently irrelevant, unhelpful, and unreliable in an evidentiary sense.

I. JEDEC's Standards and RAND Commitment

JEDEC is an international standard-setting organization (SSO) with more than 250 member companies that jointly develops standards for the microelectronics industry, including standards for solid-state devices, integrated circuits, and electronic modules, such as memory modules.[6] JEDEC's LRDIMM and RDIMM standards are widely implemented in memory modules used in the manufacture of servers.

A. *JEDEC's RDIMM, LRDIMM, DDR3, and DDR4 Standards*

Computer memory, such as the DRAM chips installed on a dual-in-line memory module (DIMM), enables a device (such as a smartphone, a computer, or a server) to store and access information actively used to run programs and perform numerous functions.[7] Some programs and functions require greater memory than others. For example, a smartphone uses less memory opening Google Maps and obtaining directions to a local restaurant than a server uses tracking the daily transactions of stocks on the S&P 500. For applications demanding significant memory, the server must be able not only to store an enormous amount of information (as measured by the server's memory capacity), but also to access that information quickly and efficiently (as measured by the server's memory bandwidth).[8] To increase both the server's memory capacity and its memory bandwidth, the user can install a memory module with a buffering interface that organizes and directs

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[6] JEDEC, JEDEC Manual of Organization and Procedure § 1.2, at 1 (July 2015), http://www.jedec.org/sites/default/files/JM21R.pdf [hereinafter JEDEC Manual]; *see also About JEDEC*, JEDEC, https://www.jedec.org/about-jedec.

[7] *See, e.g.*, *The Role of Memory in Your Computer*, Crucial, http://www.crucial.com/usa/en/support-what-does-computer-memory-dram-do.

[8] *See* Douglas Maltech, *LRDIMM vs RDIMM: Signal Integrity, Capacity, Bandwidth*, EDN Network (Aug. 1, 2014), http://www.edn.com/design/designcon/4432983/LRDIMM-vs-RDIMM--Signal-integrity--capacity--bandwidth.

the flow of information between the computer's memory controller and the computer's memory chips.[9] JEDEC's LRDIMM and RDIMM standards each specify unique interfaces for server DIMMs.

JEDEC's RDIMM standard introduces a single register on the memory module that buffers the signals sent between the DRAM chips and the server's memory controller.[10] The addition of the register on the memory module enables the user to increase the server's memory capacity relative to a DRAM module that lacks a register.[11] By adding nine distributed buffers to the DRAM module, JEDEC's DDR4 LRDIMM standard further increases the server's memory capacity while increasing the server's memory bandwidth.[12] In other words, relative to memory modules practicing JEDEC's earlier RDIMM standard, a DDR4 LRDIMM product enables greater memory capacity in a server without impeding the server's ability to operate at its highest system speed.[13] DDR4 LRDIMM products thus offer better server performance at a high memory capacity than any other server DIMM.[14]

Meanwhile, apart from the transition from the RDIMM standard to the LRDIMM standard, advances in DRAM technology have also increased the performance of memory modules. DDR4 stands for double data rate fourth-generation, which is JEDEC's standard for the latest generation of DRAM products. (JEDEC's standard for the next generation of DRAM, DDR5, is currently in development.) Relative to the earlier DDR3 products, DDR4 products offer a higher bandwidth interface.

As we will explain in Part IV below, our analysis in this article addresses the disaggregation of the incremental economic value created when server manufacturers advance from both (1) the RDIMM standard to the LRDIMM standard and (2) the DDR3 standard to the DDR4 standard. For brevity, we do not explore here the disaggregation of the incremental economic value created when server manufacturers advance from (1) practicing the DDR3 standard in conjunction with the RDIMM standard to (2) practicing the DDR4 standard in conjunction with the RDIMM standard. The methodology required for that analysis would be a simpler version of the procedure we use to disaggregate simultaneously the components of incremental value that server manufacturers derive from advancing from (1) practicing the DDR3

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[9] *See What Is Buffered Memory?*, DELL, http://www.dell.com/support/article/us/en/19/SLN39492/what-is-buffered-memory----kb-article---162872?lang=EN.

[10] Johan De Gelas, *LRDIMMs RDIMMs, and Supermicro's Latest Twin*, ANANDTECH (Aug. 3, 2012), http://www.anandtech.com/show/6068/lrdimms-rdimms-supermicros-latest-twin/2.

[11] *Id.*

[12] *See* Maltech, *LRDIMM vs RDIMM: Signal Integrity, Capacity, Bandwidth*, *supra* note 8.

[13] *Id.*; *DDR4 LRDIMMs Let You Have It All* 3–4, HEWLETT PACKARD ENTERPRISE (July 2016) [hereinafter HPE DDR4 LRDIMM White Paper], https://www.hpe.com/h20195/v2/getpdf.aspx/4AA6-6067ENW. pdf?ver=1.0.

[14] *Id.* at 2.

standard in conjunction with the RDIMM standard to (2) practicing the DDR4 standard in conjunction with the LRDIMM standard.

B. *JEDEC's Patent Policy and RAND Commitment*

Member companies that participate in JEDEC's standard-setting process must adhere to the SSO's Manual of Organization and Procedure, which defines JEDEC's patent policy and its RAND commitment.[15] JEDEC's patent policy requires members that own standard-essential patents to disclose those patents to JEDEC and memorialize their willingness (or unwillingness) to offer to license those patents on RAND terms in a license disclosure that JEDEC subsequently makes available to other members.[16]

Specifically, JEDEC requires each member, as a condition of participating in JEDEC's standard-setting process, to disclose patents and patent applications "that are owned or controlled by that . . . Member" and that the member "reasonably believe[s] . . . to contain one or more Essential Patent Claims."[17] JEDEC defines an Essential Patent Claim as one "the use of which would necessarily be infringed by the use, sale, offer for sale or other disposition of a portion of a product . . . to be compliant with the required portions of a final approved JEDEC Standard."[18]

Furthermore, JEDEC's patent policy requires the member to memorialize its disclosed essential patents and its willingness to license those patents on RAND terms in either a license assurance (for willing licensors) or a form named, "Notice of Refusal to Offer Licenses on RAND Terms" (for unwilling licensors).[19] The patent policy "applies equally to situations involving the Essential Patent Claims that are discovered after adoption of the Standard."[20] JEDEC provides its members online access to all submitted license assurances.[21]

JEDEC gives no guidance on how to determine whether a member's offer to license its SEPs to an implementer of a JEDEC standard is RAND. JEDEC's patent policy explains that "each Committee Member, as a condition of Participation, agrees to offer to license on RAND terms, to all

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[15] JEDEC Manual, *supra* note 6, § 8.2, at 24.

[16] *Id.* § 8.2.2.1.

[17] *Id.* §§ 8.2.3, 8.2.1.

[18] *Id.* § 8.2.1, at 23.

[19] *Id.* § 8.2.3; *see also* JEDEC, License Assurance/Disclosure Form, http://www.jedec.org/sites/default/files/License_Assurance-Disclosure_Form_20150710.pdf [hereinafter JEDEC License Assurance]; JEDEC, Notice of Refusal to Offer Licenses on RAND Terms Form, https://www.jedec.org/sites/default/files/Notice_Refusal-to-Offer%20Licenses_Rand-Terms_Form.pdf.

[20] JEDEC Manual, *supra* note 6, § 8.2.8, at 28.

[21] JEDEC members can access these assurances by logging on to JEDEC's member website, clicking on the "Patents" tab, and downloading an excel spreadsheet that itemizes the submitted assurances and includes embedded hyperlinks to a PDF download for each document.

Potential Licensees, such . . . Member's Essential Patent Claims."[22] Similarly, the license assurance that a member gives to JEDEC contains the following statement concerning the making of a RAND offer to a potential licensee: "A license will be offered to applicants desiring to utilize the license for the purpose of implementing the JEDEC Standard under reasonable terms and conditions that are demonstrably free of any unfair discrimination."[23]

New York law controls the interpretation of JEDEC's patent policy and the precise obligations arising from a member's RAND commitment to JEDEC.[24] In turn, the New York Court of Appeals has said that, "when parties set down their agreement in a clear, complete document, their writing should as a rule be enforced according to its terms."[25] Consequently, "[e]vidence outside the four corners of the document as to what was really intended but unstated or misstated is generally inadmissible to add to or vary the writing."[26] Under New York law, extrinsic evidence is appropriate to consider in interpreting a contract's terms only when the court has found ambiguity in the written contract.[27]

Neither JEDEC's patent policy nor the text of its license assurance addresses the SEP holder's right to seek an exclusion order from the ITC, or an injunction from a court, against a third party unwilling to enter into a license to practice patents essential to implementing a JEDEC standard. Regarding dispute resolution, JEDEC's patent policy says only that members "are encouraged, but *not required*, to bring Patent Policy issues or concerns with respect to . . . the licensing of Essential Patent Claims to the attention of the JEDEC Board of Directors for resolution."[28] In particular, neither JEDEC's patent policy nor the text of its license assurance requires JEDEC members to resort to arbitration, mediation, or any other dispute-resolution mechanism to resolve disputes regarding the licensing of their SEPs.

Furthermore, there is no evidence that, as a matter of contract interpretation, JEDEC's RAND commitment works an implicit waiver by the SEP holder of its right under federal statutory law to seek an injunction or exclusion order. JEDEC's RAND commitment is silent on whether the SEP holder waives its statutory right to seek an injunction by entering into the RAND commitment. Typically, contractual silence is not a sufficient basis

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[22] JEDEC Manual, *supra* note 6, § 8.2.4, at 26.

[23] This language appears on page 2 of JEDEC's license assurance, a specimen of which appears as Appendix II, *infra*.

[24] JEDEC Manual, *supra* note 6, § 8.2.10, at 29.

[25] WWW Assocs., Inc. v. Giancontieri, 77 N.Y.2d 157, 162 (1990).

[26] *Id.*

[27] *Id.* at 162–63. Extrinsic evidence may not be considered to create an ambiguity in the agreement. *See, e.g.*, Intercontinental Planning, Ltd. v. Daystrom, Inc., 24 N.Y.2d 372, 379 (1969) ("[I]t is equally well settled that extrinsic and parol evidence is not admissible to create an ambiguity in a written agreement which is complete and clear and unambiguous upon its face." (citations omitted)).

[28] JEDEC Manual, *supra* note 6, § 8.2.2.1, at 24 (emphasis added).

for concluding that a provision is ambiguous.[29] When a contractual provision is unambiguous, New York law follows the conventional rule of examining only evidence within the "four corners of the document."[30] Even if contractual silence were to imply ambiguity, New York courts nonetheless have said that a party to a contract "will not be deemed to have waived its statutory rights . . . without an explicit agreement between the parties or compelling evidence that the [party alleged to have made the waiver] made a conscious decision to do so."[31] Moreover, the Supreme Court of the United States has said that it "will not infer from a general contractual provision that the parties intended to waive a statutorily protected right unless the undertaking is 'explicitly stated.' More succinctly, the waiver must be clear and unmistakable. . . . [T]o waive a statutory right the duty must be established clearly and unmistakably."[32]

Consequently, there exists no evidence that the SEP holder waives its right to seek an injunction or exclusion order after it enters into JEDEC's RAND commitment. Indeed, there is evidence to the contrary: JEDEC's members reserve the right to amend the SSO's bylaws and policies pursuant to the procedures contained in the JEDEC manual.[33] In 2015, members of JEDEC discussed the possibility of changing JEDEC's patent policy to limit the SEP holder's right to seek an injunction or exclusion order.[34] However, JEDEC decided not to include such a provision in its patent policy, and no such provision exists in JEDEC's latest manual outlining its patent policy.[35]

II. The Logic of Hedonic Prices

In his seminal article from 1974, Sherwin Rosen wrote: "Hedonic prices are defined as the implicit prices of attributes and are revealed to economic agents from observed prices of differentiated products and the specific amounts of characteristics associated with them."[36] "Econometrically," he explained, "implicit prices are estimated by the first-step regression analysis

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[29] *See, e.g.*, Greenfield v. Philles Records, Inc., 98 N.Y.2d 562, 572 (2002) ("[S]ilence does not equate to contractual ambiguity."); Schmidt v. Magnetic Head Corp., 97 A.D.2d 151, 157 (N.Y. App. Div. 1983) ("An omission or mistake in a contract does not constitute an ambiguity."); Reiss v. Fin. Performance Corp., 97 N.Y.2d 195, 199 (2001).

[30] WWW Assocs., Inc. v. Giancontieri, 77 N.Y.2d 157, 162 (1990).

[31] Consedine v. Portville Cent. Sch. Dist., 12 N.Y.3d 286, 294 (2009).

[32] Metropolitan Edison Co. v. NLRB, 460 U.S. 693, 708–09 (1983) (quoting Mastro Plastics Corp. v. NLRB, 350 U.S. 270, 283 (1956)).

[33] JEDEC Manual, *supra* note 6, § 1.8, at 7 ("Modifications to this manual may be made in whole or in part, upon approval by the Board by the normal ballot process."). JEDEC outlines its balloting process in JEDEC Manual, *supra* note 6, § 6, at 18–20.

[34] *See* JEDEC Presentation, Governance Committee Report 7–8 (Feb. 9, 2015).

[35] *See* JEDEC Manual, *supra* note 6.

[36] Rosen, *supra* note 5, at 34.

(product price regressed on characteristics) in the construction of hedonic price indexes."[37] Rosen provided this intuitive explanation of hedonic prices:

> When goods can be treated as tied packages of characteristics, observed market prices are also comparable on those terms. The economic content of the relationship between observed prices and observed characteristics becomes evident once price differences among goods are recognized as equalizing differences for the alternative packages they embody.[38]

Rosen traced the theoretical foundations of hedonic prices to Kelvin Lancaster,[39] Hendrik Houthakker,[40] Richard Muth,[41] and Nobel laureates Jan Tinbergen[42] and Gary Becker.[43] Rosen described the insights of his predecessors this way:

> The spirit of these recent contributions is that consumers are also producers. Goods do not possess final consumption attributes but rather are purchased as inputs into self-production functions for ultimate characteristics. Consumers act as their own "middlemen," so to speak.[44]

Rosen contrasted this view with his own, which "interposes a *market* between buyers and sellers."[45] Distinguished economists who subsequently used Rosen's framework have included Nobel laureate James Heckman[46] and Harvard professor Ariel Pakes.[47]

Hedonic models were developed in the early 20th century to calculate real estate rental values on the basis of housing characteristics,[48] and such models were perhaps used even earlier to estimate the value of other commodities. Academic economists refined the technique and now use hedonic models in a variety of applications. For example, many governments around the world

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[37] *Id.*

[38] *Id.* at 54.

[39] Kelvin J. Lancaster, *A New Approach to Consumer Theory*, 74 J. POL. ECON. 132 (1966).

[40] H.S. Houthakker, *Compensated Changes in Quantities and Qualities Consumed*, 19 REV. ECON. STUD. 155 (1952).

[41] Richard F. Muth, *Household Production and Consumer Demand Functions*, 34 ECONOMETRICA 699 (1966).

[42] Jan Tinbergen, *On the Theory of Income Distribution*, 77 WELTWIRTSCHAFTLICHES ARCHIV 155 (1956).

[43] Gary S. Becker, *A Theory of the Allocation of Time*, 75 ECON. J. 493 (1965).

[44] Rosen, *supra* note 5, at 36.

[45] *Id.* (emphasis in original).

[46] Ivar Ekelund, James J. Heckman & Lars Nesheim, *Identification and Estimation of Hedonic Models*, 112 J. POL. ECON. S60 (2004).

[47] Ariel Pakes, *A Reconsideration of Hedonic Price Indexes with an Application to PC's*, 93 AM. ECON. REV. 1578 (2003). For a survey of the economic literature on hedonic prices, see Lars Nesheim, *Hedonic Prices*, *in* THE NEW PALGRAVE DICTIONARY OF ECONOMICS 899 (Lawrence E. Blume & Steven N. Durlauf eds., Palgrave Macmillan 2d ed. 2008).

[48] *See, e.g.*, Adrian Ball & Andrew Allen, *The Introduction of Hedonic Regression Techniques for the Quality Adjustment of Computing Equipment in the Producer Prices Index (PPI) and Harmonised Index of Consumer Prices (HICP)* (Off. of Nat'l Stat., Econ. Trends No. 592, 2003), http://webarchive.nationalarchives.gov.uk/20160105160709/http://www.ons.gov.uk/ons/rel/elmr/economic-trends--discontinued-/no--592--march-2003/index.html.

use hedonic models to make quality adjustments to inflation measures, particularly for products subject to rapid technological change, although hedonic models are used in categories ranging from cars to clothing. The best candidates for this type of analysis are goods whose component features change frequently in ways that one can easily identify and quantify.[49]

In the *Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes: Special Application to Information Technology Products*, Jack Triplett in 2004 traced the evolution of the hedonic price model as follows:

> Although Andrew Court (1939) published the first article on hedonic price indexes, researchers before Court discovered relations that resemble hedonic functions to some extent. Examples are Waugh (1928), who estimated price-characteristics functions on vegetables, and Haas, who even earlier estimated land price-location functions, as discussed in Colwell and Dilmore (1999). Taylor (1916), discussed as another precursor by Ethridge (2002), investigated quality dispersion in the cotton market and associated price differentials, but he did not relate price differentials to the characteristics of cotton in a statistical analysis.[50]

Since these early works, economists have used hedonic price models to determine the prices of various features in a product. Because hedonic models have been used to produce national data sets for almost 50 years, their use underlies much economic and business research, even when hedonic regressions have not been estimated directly within the paper. Triplett observed:

> The first government application of the characteristics price method was the New House Price Index, which has been constructed by the US Census Bureau since 1968. This index was introduced in the US national accounts beginning in 1974, and extended back to 1968. Thus, the hedonic index for new house construction is not only the first hedonic index in any country's economic statistics, it is also the first hedonic index used in any country's national accounts. This index is still published, and can be retrieved at the US Census Bureau website (US Census Bureau, undated).[51]

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[49] *See, e.g.*, Mary Kokoski, Keith Waehrer & Patricia Rozaklis, *Using Hedonic Methods for Quality Adjustment in the CPI: The Consumer Audio Products Component* 1 (Bureau of Labor Stat., Working Paper No. 344, 2001).

[50] Jack Triplett, Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes: Special Application to Information Technology Products 87 (OECD Sci., Tech. & Indus. Working Papers, No. 2004/09, 2004) (citing Andrew T. Court, *Hedonic Price Indexes with Automotive Examples*, *in* The Dynamics of Automobile Demand 99 (1939); Frederick V. Waugh, *Quality Factors Influencing Vegetable Prices*, 10 J. Farm Econ. 185 (1928); Peter Colwell & Gene Dilmore, *Who Was First? An Examination of an Early Hedonic Study*, 75 Land Econ. 620 (1999); Fred Taylor, *Relation Between Primary Market Prices and Qualities of Cotton*, U.S. Dep't Agric. Bull. No. 457 (Nov. 24, 1916); Don E. Ethridge, *Daily Hedonic Price Analysis: An Application to Regional Cotton Price Reporting*, presented at Center for European Economic Research (ZEW) conference: Price Indices and the Measurement of Quality Changes (2002)).

[51] *Id.* at 88.

Jan de Haan has subsequently observed that the use of hedonic models has expanded from housing markets to other products, particularly those in industries facing rapid technological change: "Hedonic regression has now become one of the standard tools for statistical agencies to adjust their CPIs for quality changes in markets with a high turnover of differentiated models such as PCs."[52] Allan Shampine noted that hedonic models could be used as evidence in litigation, and that the data available from statistical agencies are neutral and cover parties not directly involved in a legal action.[53] A search of the term "hedonic" in the American Economic Association's EconLit database in 2017 returned 3,251 citations to written economic works. We now employ this established empirical method to determine a reasonable royalty for a patent.

III. Estimating Hedonic Prices to Determine the Value That a Patented Invention Creates

The role of hedonic prices as economic evidence in patent-infringement litigation is to identify the value of a patented feature. One identifies the patent's value by comparing the prices and features among similar products to determine the specific contribution of the patented technology to the overall value of the infringing product, thus revealing the value that the patent adds to the price that consumers actually pay for the infringing product.

By regressing the product's total price on the product's characteristics, one determines how much value each listed component adds to the product's total value. The implicit price of each of the product's characteristics—how much the consumer values the characteristic and, therefore, how much the consumer reveals that he would be willing to pay for that characteristic—is determined statistically from observed prices in the market.[54] Thus, one can account for and measure the value that consumers attach to each qualitative component, even if the mix of components changes.

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[52] Jan De Haan, *Comment on "Hedonic Imputation Versus Time Dummy Hedonic Indexes", in* Price Index Concepts and Measurement 196 (W. Erwin Diewert, John S. Greenlees & Charles R. Hulten eds., 2009). In a critique of the Bureau of Labor Statistics' price index calculations, Jerry Hausman describes several potential issues with the use of hedonic estimation in the context of calculating a cost-of-living price index, an application that requires estimating a consumer's utility function. He argued that hedonic analysis inadequately accounted for the effects of the introduction of new products, quality changes, and efficiencies from new sales channels. He also argued that the appropriateness of hedonic analysis depends on the particular features of a product and its market. *See* Jerry Hausman, *Sources of Bias and Solutions to Bias in the Consumer Price Index*, 17 J. Econ. Persp. 23 (2003). We believe that Hausman's concerns do not apply to our analysis. Rather than calculate a price index, our analysis examines the appropriateness of a RAND rate as calculated from a consumer's demonstrated willingness to pay. The standardized, commodified nature of the products in our analysis is such that this market mitigates Hausman's critiques.

[53] Allan L. Shampine, *Price Indexes, Hedonic Analysis, and Patent Damages*, 5 J. Intell. Prop. L. & Prac. 84 (2010).

[54] *See, e.g.*, Wooldridge, *supra* note 1, at 135.

The theory of hedonic prices posits that consumers select goods based on their characteristics.[55] A heterogeneous good such as a computer or a house is valued as a bundle of individual parts, such as rooms or megabytes of disk storage, which are priced consistently even as the number and quality of the components in each bundle changes. The consumer then purchases the affordable product that is closest to his optimal bundle of characteristics. Put differently, the consumer selects not necessarily the cheapest option or his favorite option, but the product that yields the highest positive surplus, where surplus is defined, as it is defined conventionally in microeconomic theory, as "the difference between the customer's willingness to pay and the price of each product."[56]

A. *Statistical Assumptions*

Hedonic models employ the usual statistical assumptions underlying ordinary least squares (OLS) regression analysis, a statistical technique that identifies the relationship between a dependent variable and one or more independent variables in a data sample. An OLS regression estimates a relationship between each independent variable and the dependent variable, holding the other independent variables constant. The regression estimates the magnitude and the direction of change in the dependent variable that results from a one-unit increase in the independent variable. For example, an OLS regression of car price (the dependent variable) on model year (the independent variable) might find that a one-year increase in the model year predicts a $2,000 increase in the car's sales price.[57] Economists frequently use the OLS regression model in applied research and in expert economic testimony before courts and other tribunals.[58]

In addition to making the usual assumptions of OLS models generally, a hedonic regression model further assumes that consumers know the features of the product they are purchasing and that the quality of the measurement of those features is accurate. These assumptions are surely met for the products we study here—memory modules used as inputs by manufacturers of enterprise servers. These memory modules are standardized products with well-publicized prices and features, as we will make clear in our discussion below of the data that we use for our econometric analysis. The typical

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[55] *See* Rosen, *supra* note 5, at 34; Pakes, *supra* note 47, at 1580; Nesheim, *supra* note 47, at 1.

[56] Robert L. Phillips, Pricing and Revenue Optimization 56 (Stanford Univ. Press 2005).

[57] *See, e.g.*, James H. Stock & Mark W. Watson, Introduction to Econometrics 107–12, 186–89 (Addison-Wesley 3d ed. 2011).

[58] *See, e.g.*, Daniel Rubinfeld, *Reference Guide on Multiple Regression*, *in* Reference Manual on Scientific Evidence 303 (Federal Judicial Center 3d ed. 2011). For a jurist's explanation of the proper use of regression analysis in a commercial dispute, see Judge Richard Posner's extended discussion in ATA Airlines, Inc. v. Fed. Express Corp., 665 F.3d 882, 889–91 (7th Cir. 2011).

consumer is a sophisticated manufacturer of enterprise servers, such as Dell or Hewlett Packard Enterprises.

In addition, researchers performing hedonic price regressions must be aware of market limitations and possible multicollinearity in their data to ensure that the implicit comparisons in the estimating equation are valid. We show not only how careful construction of an econometric model can avoid these issues, but also that common critiques of the hedonic regression model, such as a desire to impose too much structure on market dynamics, can lead one to create the very problems one purports to solve.

As in any regression analysis, the usefulness of the results from a hedonic price regression depends upon the correct specification of the model. Several features of using hedonic analysis to estimate the value of an industry standard (from which one subsequently will identify the share of that value that a specific patent contributes) require special attention to determine the correct specification. To satisfy the Federal Circuit's requirement to net out the value of standardization from the value of a particular standard,[59] an econometric analysis of standardized products should compare the relative value of standards. Therefore, it is important that the data and model correctly identify the comparative "base case"—the standard that is "next-best" relative to the target standard. In other words, one should construct the hedonic regression model to identify the value of the next-best standard as the constant term of the regression. The coefficients on the other terms representing each product feature then measure the value that each feature contributes above and beyond the value of the next-best standard. In other words, one can calculate the relative value of product features, including the target standard, as an addition to or deduction from the value of this base case. In this way, the usefulness of the hedonic price analysis is determined by careful construction of the multivariate regression so that the value added by the standard and the product's other features is compared to a base case that already includes the value of standardization. The incremental value attributable to the new standard and other features is thereby directly identified. When the next-best standard and the target standard are closely related, the hedonic regression will most cleanly identify the incremental contribution of the new standard above the old one.

In the course of measuring the value of standardization, hedonic price analysis can account for technology in the public domain. Some might argue that a significant portion of a standard's value exists in the public domain, such as the value that one continues to derive today from basic components invented long ago, such as transistors or capacitors. However, the older, benchmark standard is likely to contain these basic components as well—and

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[59]   Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1233 (Fed. Cir. 2014).

it surely does in the case of RDIMM vis-à-vis LRDIMM. Consequently, the value of those components in the public domain already would be included in the value of the benchmark standard, which is identified by the constant term in the hedonic regression. For there to be some residual value from public intellectual property that is otherwise uncounted, one would need to identify that IP as some public technology that is incorporated in the new standard but not in the older, benchmark standard. A technical expert on the standards should be able to identify the presence and extent of such public technology, if any exists (which we doubt).

It is unlikely that a current standard, especially one in a rapidly changing industry where successive generations of standards are developed in a shorter time than the length of a patent grant, will incorporate unique, yet unpatented technology. The World Intellectual Property Organization (WIPO) has said that "it is not uncommon that the best technology for a technical standard is a proprietary technology, protected by one or more patents."[60] The ability to incorporate one's own technology into a standard is itself a reason for firms to improve on publicly available technologies. In 2016, an EU Commission published a report from IPlytics stating that "the prospect to include their patented technology into technological standards is an important incentive for firms to increase their investment in standardization. Also, patent holders have a stronger private interest to invest in improvements of existing standards if they can recoup the costs through licensing fees."[61] These considerations make it doubtful that the public domain was much larger and more valuable when JEDEC finally approved its LRDIMM standard than when JEDEC finally approved its earlier RDIMM standard. Nonetheless, for sake of completeness we include in our apportionment analysis in Part V.F the expired LRDIMM SEPs as a proxy for public-domain technology that is arguably embodied in JEDEC's LRDIMM standard but not its RDIMM standard.

B.   *The Appropriate Functional Form*

After determining the appropriate technical standards for comparison, one must determine the appropriate functional form for the hedonic price regression. We use an "additive" model, whose basic form Equation 1 specifies:

$$Price = \alpha + \beta_1 X_i + \beta_2 (Standard),\qquad\qquad (1)$$

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[60]   Esteban Burrone, *Standards, Intellectual Property Rights (IPRs) and Standards-Setting Process*, World Intell. Prop. Org. (undated) (emphasis removed), http://www.wipo.int/sme/en/documents/ip_standards_fulltext.html

[61]   Tim Pohlmann & Knut Blind, Landscaping Study on Standard Essential Patents (SEPs) 7 (2016), http://www.iplytics.com/general/eu-commission-publishes-landscaping-study-seps-iplytics/.

where *Price* is the price of the standardized product, and α is the constant that measures the value of the "base case." The terms represented by X_i are the various levels of the product's features, and the β_1 terms are the values that consumers place on the levels of those features. Finally, the term β_2 identifies the value of the target standard—that is, the value of the patented technologies that allow the standard to operate.

Other functional forms that economists commonly use in hedonic price analysis are less useful in the comparison of technical standards. The additive model has the advantage of assigning a constant value to each feature and allowing the use of binary indicator (or "dummy") variables to account for the presence or absence of the product's various features. The additive model is the only one of the three common forms of hedonic price regression models that does so. That is, the additive form is the only model structure that satisfies the Federal Circuit's directive in *Ericsson v. D-Link* to "consider the difference between the added value of the technological invention and the added value of that invention's standardization"[62] and that identifies a constant incremental value that one can attribute to the standard.

Some economists erroneously assume that other functional forms of the hedonic price model are superior to the additive model for purposes of calculating the value of a standard. Although these forms might be appropriate for measuring price relationships in other kinds of markets, they do not produce economic evidence that helps the finder of fact to answer the legal question that the Federal Circuit in *Ericsson v. D-Link* has posed in patent-infringement litigation. The frequent critique made by these economists is that the additive model does not capture price dynamics as accurately as another model might, particularly in markets for rapidly developing products that manifest high initial prices followed by lower prices. That criticism is inapt. It mischaracterizes the task at hand as being the prediction of prices over time as closely as possible. To the contrary, the relevant task is the clean identification of what customers are actually willing to pay to have access to the standardized technology, regardless of whether they are high-willingness-to-pay customers (who make their purchases soon after the standard has been adopted) or price-constrained mass-market customers (who make their purchases once the standard has become more widespread and manufacturing processes have become more efficient).

In other common functional forms, such as the semi-log or log-log forms, the hedonic model's coefficients describe price changes as percentages or elasticities. For example, consider the semi-log model below in Equation 2:

$$\ln(Price) = \alpha + \beta_1 X_i + \beta_2(Standard), \qquad\qquad (2)$$

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[62] Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1233 (Fed. Cir. 2014).

where each term measures the same features as in Equation 1, although the interpretation of the coefficients has now changed.[63] In the semi-log model, each β coefficient determines the percentage by which the product price increases or decreases with the addition of a given feature. The drawback of this particular specification of the hedonic regression model for answering the relevant legal question that the Federal Circuit posed in *Ericsson v. D-Link* is that the specification allows the value that each feature incrementally contributes to the product (in dollar terms) to vary depending on the product's other features. For example, if the model identifies the standardized technology as increasing the product's value by 10 percent, that finding implies that the technology would add $100 to a product whose other features are valued at $1,000 but only $10 to a product whose other features are valued at $100. Although this functional form might be a useful description when technologies are complementary, this form is not useful when determining the value of a standard. To answer that legal question in a patent-infringement dispute, it is more appropriate to analyze complementarities (to the extent that they exist at all) by using a different modeling specification, such as an interaction term (which captures the differential effect of one explanatory variable on the dependent variable with respect to another explanatory variable).[64]

Another common functional form of the hedonic regression model is the log-log model, as shown below in Equation 3:

$$\ln(\textit{Price}) = \alpha + \beta_1 \ln(X_i) + \beta_2 \ln(\textit{Standard}), \qquad (3)$$

where each term measures the same features as in Equation 1, although the coefficients now measure the respective elasticities of willingness to pay with respect to each feature—that is, for each feature, the percentage increase in the willingness of customers to pay for the product that would result from a one-percent increase in the level of the given feature.[65] For determining the value of a standard, this functional form has the same drawback as the semi-log form. That is, as the value of the other features changes, the percentage value added by the presence of the standard remains the same, meaning that the value added by the standard (in dollar terms) changes. Of greater concern

[63] For an example of use of the semi-log hedonic model, see Fiona Scott Morton, Jorge Silva-Risso & Florian Zettelmeyer, *What Matters in a Price Negotiation: Evidence from the U.S. Auto Retailing Industry*, 9 Quantitative Marketing & Econ. 365, 384 (2011).

[64] For example, consider a model of housing price as a function of square footage, number of bedrooms, and an interaction term: *price = β_1 sqft + β_2 bdrms + β_3 sqft × bdrms*. The interaction term suggests that the effect of an additional bedroom on housing price depends on the size of the house, and that the effect of additional square footage on housing price depends on the number of bedrooms in the house. *See* Wooldridge, *supra* note 1, at 190–91.

[65] For examples of use of the log-log hedonic model, see Aurelia Bengochea Morancho, *A Hedonic Valuation of Urban Green Areas*, 66 Landscape & Urban Planning 35, 39 (2003); Peter C. Boxall, Wing H. Chan & Melville L. McMillan, *The Impact of Oil and Natural Gas Facilities on Rural Residential Property Values: A Spatial Hedonic Analysis*, 27 Resource & Energy Econ. 248, 257 (2005).

is the fact that the log-log model cannot accommodate binary indicator variables.[66] Consequently, the log-log specification of the hedonic regression model is useless in patent-infringement disputes for measuring the incremental value added by the presence of a new standard relative to an older standard.

C. *Included and Omitted Variables*

After choosing the appropriate functional form for the hedonic regression model, one must determine which independent variables to include. To comport with the assumptions underlying hedonic price analysis, the features that one selects as variables should be readily observable to customers and measurable in the data.[67] In addition, the features that downstream manufacturers use to advertise products practicing the patents in suit are good candidates for inclusion as independent variables, because those features are likely to be the key determinants of value for the consumer.[68] For example, if an electronics retailer advertises a 4-terabyte portable hard-drive with a USB-C connection, that information indicates that storage, portability, and port type are good candidate variables.

1. *Temporal Effects*

It is sometimes useful to include in a hedonic regression model controls for time to account for changes in the market, although the analyst must consider carefully the assumptions underlying the inclusion of those variables. For example, one might include a continuous time variable or quarter fixed effects to account for general changes in price or market conditions over time. Those controls are particularly useful when the product incorporating the new standard incorporates many of the same materials (and therefore uses similar supply chains) as does the product incorporating the older standard, as the two products will similarly respond to supply shocks or input price increases. Because the two products respond similarly, a single variable can account for market-wide effects for both.

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[66] *See, e.g.*, W. Erwin Diewert, *Hedonic Regressions: A Consumer Theory Approach*, *in* Scanner Data and Price Indexes 317, 327 (Robert C. Feenstra & Matthew D. Shapiro eds., 2003), http://www.nber.org/chapters/c9740.pdf ("[T]he semilog model has an advantage compared to the log-log model: the semilog model can deal with situations in which one or more characteristics . . . are equal to zero, whereas the log-log model cannot.").

[67] *See* Peter Kennedy, A Guide to Econometrics 107–09 (MIT Press 5th ed. 2003); Bryan K. Orme, Getting Started with Conjoint Analysis: Strategies for Product Design and Pricing Research 52 (Research Publishers 2d ed. 2010); Vithala R. Rao, Applied Conjoint Analysis 43–44 (Springer 2014).

[68] *See* Orme, *supra* note 67, at 52; Rao, *supra* note 67, at 43.

2.   *Life-Cycle Effects*

Another temporal effect for which one might control in a hedonic regression model is the presence of "life-cycle" effects—that is, a standard's tendency to lose value over time. To control for that tendency, one might wish to include a variable in the hedonic regression measuring the time since the standard was adopted. However, including such a variable would impose the additional, unfounded assumption on the model that all standards and product features decline in value at the same rate and that there is a deterministic, long-term structural price trend. If prices are not determined by a predictable decline over time, then this assumption would be inappropriate and would generate misleading results. If a newer standard is more durable and loses value more slowly than the older standard, then forcing on the data this assumption of a constant rate of decay would misrepresent the value of the more durable standard.

This situation shows how an expert witness using hedonic price analysis must understand the effects of all his assumptions of the functional form lest he mistakenly cause the very problem that he purports to solve. For example, in attempting to control for a decline in prices, imposing life-cycle effects can force two different standards to respond in similar ways, an assumption that the data might not substantiate. In such a scenario, the regression would undervalue the new standard.

Although the additive model that we use is not immune to this issue, it has the virtue of being a nonparametric model, meaning that it "use[s] data to infer an unknown quantity while making as few assumptions as possible."[69] For example, our hedonic model is nonparametric because the dummy variables take their appropriate values, as determined by the data, without being constrained by fitting the curve of a particular mathematical function. The fact that the additive version of the hedonic regression model does not impose a particular statistical relationship between the levels of the product's features is one well-founded reason for its popularity in scholarly research.[70]

3.   *Producer Costs*

It is inappropriate to include producer costs directly in a hedonic price model, as customers do not observe those costs and do not predicate their consumption decisions on them. If producer costs differ between standards, one should properly include the cost difference elsewhere in the analysis, outside the hedonic regression. However, one must take care to define

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[69]   Larry Wasserman, All of Nonparametric Statistics 1 (Springer 2006).

[70]   *See, e.g.*, Rao, *supra* note 67, Appendix at 31–33 (listing a sample of applications of conjoint analysis); Orme, *supra* note 67, at 3 ("[A] simple additive model . . . tends to work well in practice.").

appropriately the relevant costs before commencing such an analysis. After all, "[w]ith any exposure to the field of accounting one will quickly realize that the simple question 'How much did this product cost?' usually does not have a simple answer."[71]

Anyone contending that a hedonic price analysis is flawed because it ignores a pertinent element of producer cost must first define clearly and then apply the proper measure of production cost to use in this context, which is *incremental cost*.[72] Incremental cost has a formal economic definition: "if $x, y, z, \ldots$ represents the outputs of the firm's various products, and $TC(x, y, z, \ldots)$ is the total cost that the firm must incur to produce that combination of outputs, then the incremental cost of X is $IC_x = TC(x, y, z, \ldots) - TC(0, y, z, \ldots)$."[73] One typically divides this change in the firm's total cost by its level of output to express its incremental cost on a per-unit basis: "Incremental cost is a generic concept referring to the addition, per unit of the additional output in question, to the firm's total cost when the output of X expands by some preselected increment."[74] For purposes of our analysis, incremental cost is the difference between the total cost that the producer incurs by implementing a given standard and the total cost that the producer incurs without implementing the standard, divided by that producer's number of units of output practicing the new standard. One can then deduct this incremental cost from the per-unit value of the new standard to derive the net value per unit that a licensee derives from implementing that new standard.[75]

In addition to defining the proper incremental cost, one must avoid multicollinearity—that is, including multiple independent variables that measure the same effects.[76] The inclusion of collinear variables can "overfit" the hedonic price regression, which can reduce its predictive accuracy outside the sample.[77] For example, it might not be possible to include both a life-cycle effect and another time variable in the hedonic price regression to account for supply or cost changes, because they would measure the passage of time in the same way.

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[71] Phillips, *supra* note 56, at 60.

[72] *See, e.g.*, *id*. ("Accountants have derived a host of product-costing methodologies, including fully allocated costs, partially allocated costs, marginal costs, avoided costs, and financial costs, to name a few. While each costing methodology has its place, pricing and revenue optimization decisions are just based on the *incremental cost* of a customer commitment." (emphasis in original)).

[73] J. Gregory Sidak, *Maximizing the U.S. Postal Service's Profits from Competitive Products*, 11 J. Competition L. & Econ. 617, 622 (2015) (citing William J. Baumol & J. Gregory Sidak, Toward Competition in Local Telephony 57 (AEI Press & MIT Press 1994)).

[74] Baumol & Sidak, *supra* note 73, at 57.

[75] *See* Sidak, *The Value of a Standard Versus the Value of Standardization*, *supra* note 3, at 66–69.

[76] *See, e.g.*, Stock & Watson, *supra* note 57, at 197 (defining multicollinearity as "an inconvenient situation" in which one of the regressors is a perfect linear function of the other regressors, such that the ordinary least squares (OLS) estimator is "impossible to compute").

[77] *See, e.g.*, Orme, *supra* note 67, at 185 ("When additional parameters added to a model are not truly useful, they can harm the predictive accuracy of out-of-sample observations.").

4. Supply Factors

As with producer costs, it is inappropriate to include supply-related factors directly in a hedonic regression model, because they are irrelevant to the value that the consumer derives from the product. In addition, it is unlikely that anyone can predict supply shocks; consequently, imposing a deterministic structure on the presence or absence of supply shocks would be inappropriate. If a standard is for some reason riskier to implement—that is, if the standard is more susceptible to random market shocks—then one should account for that risk in a separate cost calculation outside the hedonic regression model. More fundamentally, one cannot adequately model truly random future supply shocks in *any* regression analysis, and one can control for the price effect of past shocks by using simple time dummy variables at the time of the shock. Long-term structural price trends face the same issues as the "life-cycle" effects that we discussed previously.

5. Interaction Variables

Interaction variables might be a superior alternative to using a semi-log form of the hedonic regression model to measure the value of complementarities or the effect of multiple standards. For example, if a hard drive incorporates both the USB-C standard and an Ethernet standard, then an interaction term could measure the additional value of including those features *together* (above and beyond the value of including each feature individually). One then would need to undertake the separate task of apportioning that complementary value between the standards. In addition, the interaction-variable approach is of course dependent on the availability of sufficient data to identify the effect of including each feature without overfitting the model.

IV. Using Hedonic Prices to Measure the Incremental Contribution of Load-Reduced Dual-Inline Memory Modules

We now use econometric methods to calculate Netlist's contribution to the net incremental value that JEDEC's LRDIMM standard adds above and beyond the earlier RDIMM standard. Our econometric analysis proceeds in two steps: (1) a hedonic regression analysis that calculates the incremental value of the LRDIMM standard, and then (2) an apportionment of the net incremental value of the LRDIMM standard across SEP holders, including the complainant in the ITC's 1023 investigation (Netlist). In this part, we report the results of the first step of our analysis.

We first analyze in layman's terms the technological difference between an RDIMM and an LRDIMM. Then, using transactional price data for representative RDIMM and LRDIMM products that have been compiled by De Dios & Associates, a market research firm, we specify a hedonic regression model to calculate the incremental value of JEDEC's LRDIMM standard above and beyond the value of JEDEC's RDIMM standard.[78] That is, we analyze the value that the LRDIMM standard adds to a given memory-module product relative to the value that the next-best available substitute for the standard (RDIMM) adds.

We find that implementing the LRDIMM standard into a given memory module increases the price that the manufacturer can charge for that memory module by approximately $100.47 per unit. After accounting for the incremental increase in input costs of an LRDIMM product relative to an RDIMM product (which we conservatively set at $20.70 per unit), we find that the LRDIMM standard enables the manufacturer of an enterprise server to gain an additional $79.77 per unit in after-cost value from selling an LRDIMM product (that is, $100.47 − $20.70).

A. *The LRDIMM Technology*

Before using hedonic regression analysis to calculate the incremental value of JEDEC's LRDIMM standard above and beyond its earlier RDIMM standard, we first examine in layman's terms the technological difference between an RDIMM and a DDR4 LRDIMM. RDIMMs and LRDIMMs are different types of memory modules commonly used in enterprise servers.[79] Typically, a DDR4 LRDIMM has a higher memory capacity and process speed relative to an RDIMM.[80] That higher capacity and speed result from the data buffer technology of a DDR4 LRDIMM,[81] which we describe using Figure 1.

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[78] The transactional prices for memory modules are a proprietary database available for purchase from De Dios & Associates. *See* https://dedios.com.

[79] *See, e.g.*, *Technical Brief*, Kingston Technology, http://www.kingston.com/us/memory/resources/lrdimms; Rambus, Server DIMM Chipsets 2 (2016), https://42xtjqm0qj0382ac91ye9exr-wpengine.netdna-ssl.com/wp-content/uploads/2016/08/Server-DIMM-Chipsets-Solution-Overview.pdf.

[80] *See Technical Brief*, Kingston Technology, *supra* note 79.

[81] *See id.*

Figure 1. Layout of the Data Transmission Circuits
and Control Circuit of a DDR4 LRDIMM
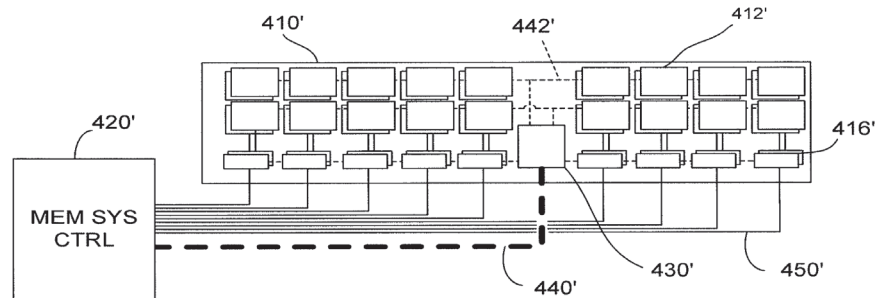
Source: U.S. Patent No. 8,516,185 fig. 3C (Apr. 15, 2010).

As Figure 1 shows, in a DDR4 LRDIMM, DRAM chips (412′) are mounted on a printed circuit board (PCB) (410′).[82] The system memory controller (420′) sends control signals through the address and control lines (440′) to the control circuit (430′), which then transmits control signals through transmission lines (442′) to the DRAM chips.[83] The DRAM chips then transmit data to the plurality of data transmission circuits comprising byte-wise data buffers (416′), which transmit data through data lines (450′) back to the system memory controller.[84] The data transmission circuits comprising byte-wise data buffers "reduce the load seen by the system memory controller," enabling DDR4 LRDIMMs to operate at a higher speed and higher capacity relative to RDIMMs in enterprise servers.[85]

Although Figure 1 illustrates a DDR4 LRDIMM, the components labeled 410′, 412′, 420′, 430′, 440′, 442′, and 450′ enable functionalities that are also present in an RDIMM.[86] Put differently, an RDIMM also consists of DRAM chips mounted on a PCB, a system memory controller, a control circuit, address and control lines, and data transmission lines.[87] In an RDIMM, however, data are transmitted directly from the DRAM chips to the system memory controller, rather than through the data transmission circuits that comprise the byte-wise data buffers.[88] Hence, the physical difference between

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[82] U.S. Patent No. 8,516,185 col. 7 ll. 61–65 (Apr. 15, 2010).

[83] *Id.* col. 9 ll. 62–65, col. 10 ll. 10–15.

[84] *Id.* col. 10 ll. 31–40.

[85] *Id.* col. 10 ll. 41–47; *see also Technical Brief*, Kingston Technology, *supra* note 79 ("LRDIMMs greatly reduce the electrical loading of the DRAM chips onto the memory bus. . . . Through the reduction of the electrical Ranks of the LRDIMM, the server is then able to support LRDIMMs at higher speeds than RDIMMs, and with fewer restrictions on [capacity]."); Ryan Smith, *Rambus to Go into Fabless Chip Production, Announces RB26 DDR4 DIMM Chipset*, AnandTech (Aug. 17, 2015), http://www.anandtech.com/show/9534/rambus-to-go-into-fabless-chip-production-announces-rb26-ddr4-dimm-chipset.

[86] *See, e.g.*, Rambus, Server DIMM Chipsets, *supra* note 79, at 2.

[87] *See id.*

[88] *See id.*

an RDIMM and a DDR4 LRDIMM arises from the presence (or absence) of the data buffer technology. Because a DDR4 LRDIMM includes additional data-buffer chips, the cost of producing a DDR4 LRDIMM is higher than the cost of producing an RDIMM for a given memory density.[89] We account for that incremental cost of production in Part IV.D below.

B.   The Price Data

To derive the value of each standard over the value of that standard's next-best available substitute—that is, the incremental value in this case of the LRDIMM standard relative to the RDIMM standard—we use De Dios & Associates' proprietary price data for purchases of representative RDIMM and LRDIMM products from 2010 to the end of 2017. These data describe the average prices that original equipment manufacturers (OEMs) paid for various configurations of memory modules in market transactions. However, only the data from 2010 to 2016 include observed market prices. Thus, we exclude from our analysis the forecasted price data for the year 2017. We also exclude data for 2010, 2011, and 2012 from our analysis because no LRDIMM products or DDR4 products (either LRDIMM or RDIMM) were sold during those years. Put differently, our regression analysis uses only price data from actual transactions occurring from 2013 through 2016.

Because the data we use spans only four years during a period of low inflation, we do not adjust the data according to the personal consumption expenditure (PCE) index to account for changes in purchasing power. For a longer time series, it would of course be appropriate to make such an adjustment.

1.   *LRDIMMs' Continued Price Premium Above RDIMMs*

A simple comparison of the average price of LRDIMM products with the average price of RDIMM products from 2013 to 2016 indicates that LRDIMM products command a significant price premium above RDIMM products. Figure 2 below shows that price relationship.

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[89]   *See* Smith, *supra* note 85.

Figure 2. Average OEM Prices for RDIMM
and LRDIMM Products, 2010 to 2016
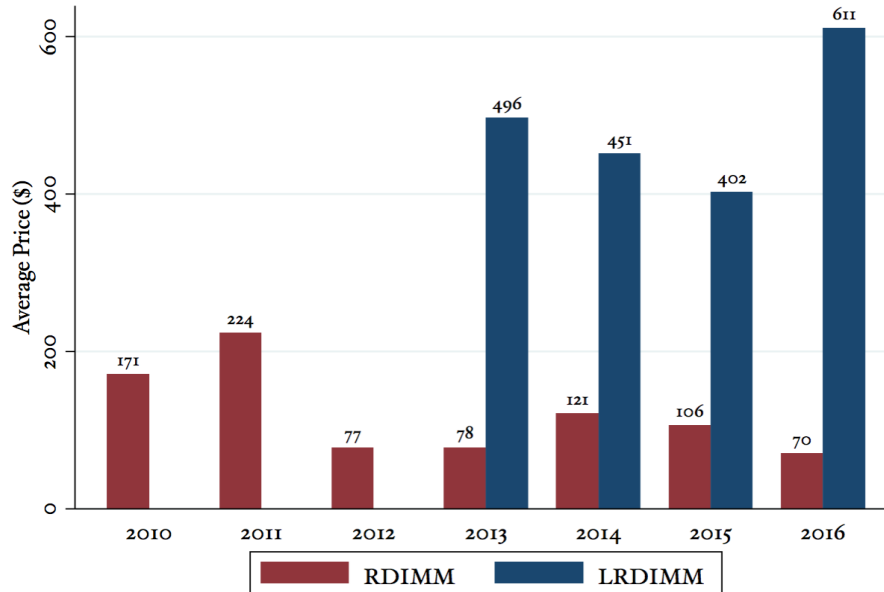
Source: De Dios & Associates (2016).
Note: DDR4 LRDIMM did not become available until 2014.

We can further examine the relative prices for LRDIMM and RDIMM products over time, while accounting for their differences in features. Figure 3 shows that this premium for LRDIMM products remains even if we analyze memory modules on a per-GB basis. Although commonly used, the per-GB basis implicitly imposes the economic requirement that a memory module's price or cost scales linearly with its capacity measured in number of gigabytes. The data might violate this often-unspoken assumption and, in fact, we find that customers often pay a premium for higher-GB memory modules.

Figure 3. Average OEM Price-Per-Gigabyte for
RDIMM and LRDIMM Products, 2010 to 2016
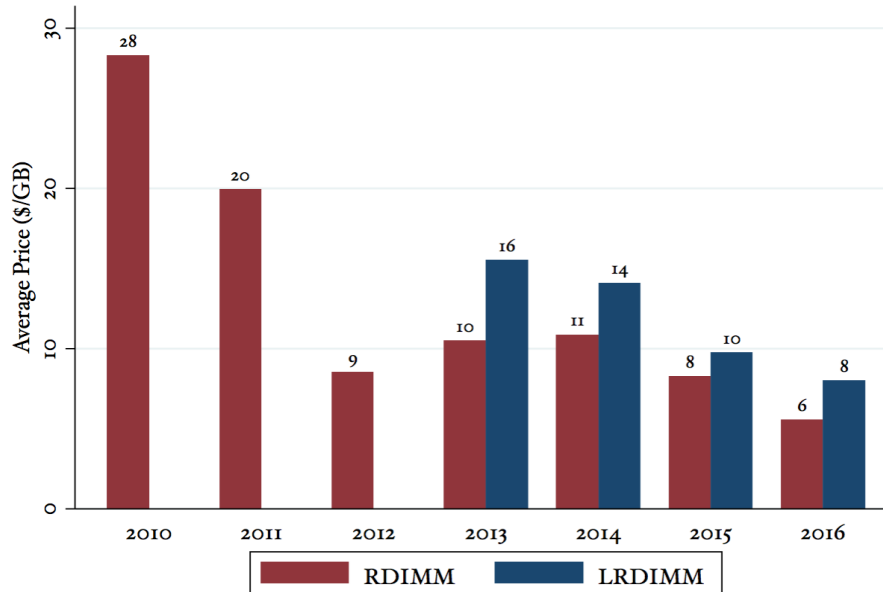
Source: De Dios & Associates (2016).
Note: DDR4 LRDIMM did not become available until 2014.

This discrepancy between common practice (with its tacit assumption about linear scaling of the willingness to pay for higher gigabyte capacity on a memory module) and the underlying data further illustrates the need for and usefulness of more robust statistical techniques than simple arithmetic calculations of average prices. However, conducting that kind of statistical analysis on a feature-by-feature basis is an inefficient use of the available data in an econometric sense. Hedonic analysis that employs a multivariate regression technique can simultaneously account for price variations due to multiple features of memory modules while controlling for changes in the market over time.

> 2. *Why* Ex Post *Data Enable Scientific Analysis of the LRDIMM Price Premium Whereas Hypothetical* Ex Ante *Data Cannot*

In our hedonic regression model, we rely exclusively on *ex post* data rather than *ex ante* data. Our reason for doing so is both simple and compelling: *ex post* data are observable, whereas *ex ante* data are not observable because they do not yet exist. Consequently, *ex post* data actually enable one to answer a testable scientific question. In contrast, reliance on (nonexistent) *ex ante* data

merely adds one person's opinion to a philosophical debate that can never be factually resolved for lack of observable data necessary to conduct empirical analysis. As the Supreme Court said in *Daubert*, quoting Sir Karl Popper's *Conjectures and Refutations*, "[t]he criterion of the scientific status of a theory is its falsifiability, or refutability, or testability."[90] A theory predicated on data that do not exist is necessarily nonfalsifiable.

We expect that the proponents of the patent-holdup conjecture, who urge courts to calculate RAND or FRAND royalties on that basis of an *ex ante* hypothetical negotiation at the time of standard adoption,[91] will disparage our decision to rely on *ex post* data. However, the alternative analysis that they propose is unscientific, both as an epistemological matter and an evidentiary matter. It lacks an empirical foundation because it would require making hypothetical valuations of the incremental value of the SEPs in suit before data from market transactions had come to exist. Those valuations would necessarily be speculative in an evidentiary sense. In contrast, sound statistical analysis rests on observed data, not conjecture. Put differently, one cannot subject a conjecture to attempts at falsification by conducting supposedly empirical analysis of data that are conjectured to exist but cannot actually be observed. The data that would be required to inform an *ex ante* analysis of a hypothetical license negotiation at the moment of standard adoption simply cannot and do not exist. We return in Part VII.C to this examination of why an *ex ante* incremental-value analysis lacks economic rigor.

## C.	A Hedonic Model to Derive the Incremental Value of JEDEC's LRDIMM Standard

Customers differentiate memory modules on the basis of product features, which include the module's gigabyte capacity (the module's total amount of storage); the module's use of a particular generation of DRAM, as denoted by its DDR type (for example, DDR3 or DDR4); and the particular JEDEC standard that the module practices (for example, RDIMM or LRDIMM). A hedonic regression model accounts for how the features included in a given

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[90] *Cf.* Daubert v. Merrill Dow Pharm., Inc., 509 U.S. 579, 593 (1993) (quoting Karl R. Popper, Conjectures and Refutations: The Growth of Scientific Knowledge 37 (5th ed. 1989) (emphasis omitted)).

[91] *See* Fiona M. Scott Morton & Carl Shapiro, *Strategic Patent Acquisitions*, 79 Antitrust L.J. 463, 471 (2014); Joseph Farrell, John Hayes, Carl Shapiro & Theresa Sullivan, *Standard Setting, Patents, and Hold-Up*, 74 Antitrust L.J. 603, 607, 624 (2007); Mark A. Lemley & Carl Shapiro, *Patent Holdup and Royalty Stacking*, 85 Tex. L. Rev. 1991, 1993, 2010 (2007); Kai-Uwe Kühn, Fiona Scott Morton & Howard Shelanski, *Standard Setting Organizations Can Help Solve the Standard Essential Patents Licensing Problem*, CPI Antitrust Chron., Mar. 2013, at 1, 3. For detailed refutations of the patent-holdup conjecture, see J. Gregory Sidak, *The Meaning of FRAND, Part I: Royalties*, 9 J. Competition L. & Econ. 931 (2013); J. Gregory Sidak, *Does the International Trade Commission Facilitate Patent Holdup?*, 1 Criterion J. on Innovation 601 (2016); Alexander Galetovic & Stephen Haber, *The Fallacies of Patent-Holdup Theory*, 13 J. Competition L. & Econ. 1 (2017).

memory-module product determine its price. A hedonic regression model can reliably estimate the implicit price that consumers demonstrate that they are willing to pay for each of those distinguished features.

There might also be economy-wide or sector-wide effects that influence the price level of memory modules from year to year. To account for these effects, we control for the year in which each price was charged.

    1.   Using a Hedonic Regression Model to Identify Customers' Demonstrated Willingness to Pay for a Memory Module's Features

We specify a hedonic regression model to analyze the effect of each feature on the price of the memory module and separately determine how much customers demonstrated that they were willing to pay for a given feature or for an average memory module at a given time. Our hedonic regression model to identify the incremental value of the LRDIMM standard is expressed in the following equation:

$$Price = \alpha + \beta_1 \times LRDIMM + \beta_2 \times DDR4 + \beta_3 \times GB + \beta_4 \times Year + \varepsilon, \qquad (4)$$

where *LRDIMM* is a dummy variable that takes a value of 1 if the memory module is LRDIMM-compliant, *DDR4* is a dummy variable that takes a value of 1 if the memory module is DDR4-compliant, *GB* is a continuous variable for the memory module's gigabyte capacity, and *Year* is a variable for each year for which we have price data. The beta coefficients (β_i) in our hedonic regression model measure the average effect of a given feature on the final price of a given memory module. For example, β_2 measures the average effect that being compatible with the DDR4 standard has on the price of a memory module. Similarly, β_1 identifies the incremental value that being compatible with the LRDIMM standard adds to a memory module. The constant term, *α*, is the base-case product. It measures the value attributable to standardization and the product's basic functionality. By using the next-best standard (namely, DDR3 RDIMM with 4 GB of capacity in 2013) as the base case, we can measure the value of standardization by carefully selecting appropriate data to make a valid comparison between similar standardized products. Table 1 reports the results of the hedonic regression on the price data.

Table 1. Hedonic Regression Results for Memory-Module Prices

Variable	Coefficient	Standard Error	*t*-Statistic
LRDIMM	100.4716***	35.0722	2.86
DDR4	48.7459***	15.8708	3.07
GB			
8	61.4273*	31.9943	1.92
16	108.5513***	34.4684	3.15
32	223.0828***	45.3401	4.92
64	577.4247***	63.9095	9.04
128	1457.388***	69.2066	21.06
Year			
2014	−9.3930	23.6438	−0.40
2015	−61.0179***	23.3763	−2.61
2016	−128.272***	23.0100	−5.57
Constant	46.6667*	25.5737	1.82
R^2	0.8981		
F-Statistic	141.01		
Prob > F	0.0000		
N	171		

Source: De Dios & Associates (2016).
Notes: * indicates statistical significance at the 90-percent confidence level, ** indicates statistical significance at the 95-percent confidence level, and *** indicates statistical significance at the 99-percent confidence level.

Table 1 reports that the patented technologies that allow the LRDIMM standard to operate add $100.47 in value above the RDIMM standard to a given memory module. The *LRDIMM* variable identifies the incremental value of the LRDIMM standard (excluding value attributable to the module's gigabyte capacity) to the DDR4 standard, and to market conditions prevailing in the year in which the memory module was sold. LRDIMM prices exceed RDIMM prices by a statistically significant amount, indicating that the *LRDIMM* variable has explanatory power and that we should include the *LRDIMM* variable in our hedonic regression model.

2. *Testing the Statistical Significance and Robustness of the Hedonic Regression Model to the Inclusion of the* LRDIMM *and* DDR4 *Variables*

Here, we discuss the statistical significance of our hedonic regression findings and confirm that we have correctly specified our model to identify the incremental value of the LRDIMM standard. The *t*-statistics that our regression

results report indicate that all of the measured features explain differences in price from the base-case product, except that, on average, prices in 2014 do not statistically differ from prices in 2013. Both the *LRDIMM* and the *DDR4* variables are statistically significant at the 99-percent confidence level, which indicates that those variables have explanatory power in our hedonic regression model.

We conduct a Wald test to analyze whether the *LRDIMM* variable and the *DDR4* variable are jointly statistically significant.[92] We obtain an *F*-statistic of 7.22 and a *p*-value of 0.0010, which indicate that the *LRDIMM* variable and the *DDR4* variable add explanatory power to our hedonic regression model with a likelihood of 99.9 percent. This result is consonant with economic reasoning and common sense because the purpose of memory modules is to store and process information in enterprise servers as quickly as possible. One would therefore expect the buyers of memory modules—namely, manufacturers of enterprise servers—to value technologies that improve the performance of memory modules. Individually, the *LRDIMM* variable has an *F*-statistic of 8.21 with a *p*-value of 0.0047, and the *DDR4* variable has an *F*-statistic of 9.43 with a *p*-value of 0.0025. These Wald-test results confirm the results of the *t*-statistics that Table 1 already reported—namely, that technologies that enable higher speeds for memory modules have meaningful explanatory power in econometric terms.

Table 1 also reports the R^2 statistic, which indicates how much of the data's variation our hedonic regression model explains. An R^2 value of 0.8981 means that a model using only the marketed features of the memory module explains almost 90 percent of the variation in observed prices among memory modules.

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[92] *See, e.g.*, Helvi Kyngäs & Marianne Rissanen, *Support as a Crucial Predictor of Good Compliance of Adolescents with a Chronic Disease*, 10 J. Clinical Nursing 767, 774 (2001) ("The Wald test is a way of testing the significance of particular explanatory variables in a statistical model. . . . If for a particular explanatory variable, or group of explanatory variables, the Wald test is significant, then we would conclude that the parameters associated with these variables are not zero, so that the variables should be included in the model. If the Wald test is not significant[,] then these explanatory variables can be omitted from the model."); *FAQ: How are the Likelihood Ratio, Wald, and Lagrange Multiplier (Score) Tests Different and/or Similar?*, UCLA Institute for Digital Research and Education, http://stats.idre.ucla.edu/ other/mult-pkg/faq/general/faqhow-are-the-likelihood-ratio-wald-and-lagrange-multiplier-score-tests-dif- ferent-andor-similar/ ("The Wald test works by testing the null hypothesis that a set of parameters is equal to some value. . . . [T]he Wald test can be used to test multiple parameters simultaneously."); *see also* Stock & Watson, *supra* note 57, at 712 ("[T]he homoscedastic-only *F*-statistic and the Wald *F*-statistic are two versions of the same statistic. That is, the two expressions are equivalent.").

3. *Using a Wald Test to Determine Whether Any Single Variable, or Any Combination of Variables, Adds Statistically Meaningful Explanatory Power to the Hedonic Regression Model*

To test the overall statistical significance of our hedonic regression model, we also perform a Wald test, from which we obtain an *F*-statistic of 141.01 and a *p*-value of 0.0000. These results enable us to reject the null hypothesis that our hedonic regression model is not statistically significant.[93] Put differently, a Wald test confirms the overall statistical significance of our hedonic regression model and indicates that observable characteristics of memory modules explain the variation in the prices actually paid for those modules. That is, customers demonstrate that they are willing to pay a higher price for improved module features. Thus, we conclude that the product features that we examine in our hedonic regression, as reported in Table 1, are the main determinants of the price for a memory module.[94]

Some might still contend that other features of memory modules explain almost as much of the variation in the data, such that our including the *LRDIMM* variable in our hedonic regression is unnecessary. We strongly disagree. This critique misunderstands the meaning of R^2.[95] The R^2 statistic does not increase linearly—that is, two single-variable models with an R^2 of 0.2 and 0.3 cannot be combined to produce one two-variable model with an R^2 of 0.5. For an economist or statistician giving expert testimony to suggest otherwise would be unscientific and misleading.

Instead, one should perform a Wald test to determine whether a single variable, or a particular combination of variables, adds statistically meaningful explanatory power to a model. The R^2 statistic can help examine the statistical significance of our hedonic regression model relative to other potential

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[93] The *F*-statistic indicates whether our hedonic regression model has any explanatory power. "This null hypothesis is, in a way, very pessimistic. It states that *none* of the explanatory variables has any effect on *y*." WOOLDRIDGE, *supra* note 1, at 148 (emphasis in original); *see also* DAVID R. ANDERSON, DENNIS J. SWEENEY & THOMAS A. WILLIAMS, STATISTICS FOR BUSINESS AND ECONOMICS 658 (South-Western 11th ed. 2011) ("The *F* test is used to determine whether a significant relationship exists between the dependent variable and the set of all independent variables.").

[94] Because it can identify the main determinants of the price paid for a memory module, our hedonic regression model has empirical relevance as well to application of the entire market value rule (EMVR). In a case that will be tried before a jury, the alleged infringer typically claims that the patent in suit does not drive demand for the patent-practicing multicomponent product, such that the royalty base should not be the net retail price of the practicing product but rather should be the price of the component within that product constituting the smallest salable patent-practicing unit (SSPPU). *See* J. Gregory Sidak, *The Proper Royalty Base for Patent Damages*, 10 J. COMPETITION L. & ECON. 989 (2014); J. Gregory Sidak, *Apportionment, FRAND Royalties, and Comparable Licenses After* Ericsson v. D-Link, 2016 U. ILL. L. REV. 1809.

[95] *See, e.g.*, Kyngäs & Rissanen, *supra* note 92, at 774; CHRISTOPHER F. BAUM, AN INTRODUCTION TO MODERN ECONOMETRICS USING STATA 98–100 (Stata Press 2006) ("Multiple restrictions on the coefficient vector imply a joint test, the result of which is not simply a box score of individual tests. Every user of regression is familiar with this concept. . . . Examining goodness of fit by comparing Root MSE [Mean-Square Error] or noting that one of these models has a higher R^2 . . . is not likely to yield conclusive results and lacks a statistical rationale." (emphasis removed)).

formulations. However, more information is necessary to make this comparison than a visual comparison of the R^2 in two different regressions. Common statistical tests exist to allow a rigorous comparison.

A Wald test can also establish whether the model we present provides more explanatory power on the determinants of prices than a competing model.[96] A Wald test indicates that both the *LRDIMM* variable, as well as the combination of the *LRDIMM* and *DDR4* variables, should be included in our hedonic regression model, even though the addition of those variables increases the R^2 only slightly from similar single-variable models. Table 2 reports these results. As we have explained above, the *F*-statistic for joint significance of the *LRDIMM* and *DDR4* variables indicates that a model that includes these variables is more informative than a model that neglects these variables and instead examines only memory capacity and year. In economic terms, these empirical findings indicate that customers of memory modules care about more than merely the *amount* of data they can store—they care also about their ability to access that data *quickly*.

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[96] *See, e.g.*, Baum, *supra* note 95, at 100 ("How do we compare two regression models that attempt to explain the same response variable but that differ in their regressor lists? If one of the models is strictly nested within the other, we can . . . apply a Wald test to the original or unconstrained model to evaluate whether the data reject the restrictions implied by the constrained model. This approach works well for classical hypothesis testing where the parameters of one model are a proper subset of another.").

Table 2. Hedonic Regression Results for
Memory-Module Prices and Wald Test Results

	[1]	[2]	[3]	[4]	[5]
LRDIMM	100.4716*** (35.0722)	418.1868*** (37.24445)			
DDR4	48.74588*** (15.87077)		165.3677*** (39.41668)		
GB					
8	61.42727* (31.9943)			24.85612 (32.01517)	
16	108.5513*** (34.46838)			65.52494* (34.17588)	
32	223.0828*** (45.34008)			265.9794*** (34.06557)	
64	577.4247*** (63.90946)			620.7882*** (51.43789)	
128	1457.388*** (69.20657)			1478.333*** (59.39536)	
Year					
2014	−9.392964 (23.6438)				56.34597 (63.12906)
2015	−61.01791** (23.37634)				40.28006 (61.24793)
2016	−128.272*** (23.00999)				88.55171 (57.60612)
Constant	46.66667* (25.57373)	91.69456*** (17.78672)	110.6726*** (26.7914)	46.66667 (29.69768)	135.9859*** (44.94369)
R^2	0.8981	0.4273	0.0943	0.8583	0.0145
F-Statistic	141.01	126.07	17.60	199.86	0.82
Prob > F	0.0000***	0.0000***	0.0000***	0.0000***	0.4855
N	171	171	171	171	171

Source: De Dios & Associates (2016).
Notes: * indicates statistical significance at the 90-percent confidence level, ** indicates statistical significance at the 95-percent confidence level, and *** indicates statistical significance at the 99-percent confidence level. The Wald test results shown in Table 2 (that is, the *F*-statistic and the *Prob > F* reported for each specification) confirm the overall significance of the variables for each regression except for the fifth regression.

Table 2 reports the results of our hedonic regression model, which measures the effect of each feature of a memory module—such as module type

(RDIMM or LRDIMM), generation of DRAM (DDR3 or DDR4), and giga-byte capacity—on a memory module's price. To measure the extent to which each feature of a memory module explains the relative variation in the price data, we ran four separate regressions and observed the R^2 values of each regression. To examine whether each feature provided explanatory power, we show the F-statistic for the overall statistical significance of each of the four regressions. Year values, by themselves, do not have significant statistical power in explaining the prices paid for memory modules. However, we include those variables because they are statistically significant when combined with other explanatory variables, as column 1 of Table 2 indicates.

Column 2 in Table 2 reports the results of our regression of the price data on the *LRDIMM* indicator variable, which equals 1 if the module is LRDIMM-compliant and equals 0 if the module is only RDIMM-compliant. An R^2 of 0.4273 indicates that the *LRDIMM* variable accounts for 42.73 percent of the variation in the price data.

Column 3 reports the results of our regression of the price data on the *DDR4* indicator variable, which equals 1 if the module is compatible with DDR4 memory and equals 0 if the module is compatible only with DDR3 memory. An R^2 of 0.0943 indicates that the *DDR4* variable accounts for 9.43 percent of the variation in the price data.

Column 4 reports the results of our regression of the price data on the *GB* categorical variable, which indicates the memory module's gigabyte capacity from 8GB to 128GB. An R^2 of 0.8583 indicates that the *GB* variable accounts for 85.83 percent of the variation in the price data.

Column 5 reports that the year in which the module's price was measured explains only 1.45 percent of the variation in the price data.

Among all the independent variables, a module's gigabyte capacity individually explains the highest amount of variation in the price data. However, the R^2 values of individual variables in a multivariate regression model are not simply additive. R^2 values of course range from 0 to 1.[97] Put differently, taking the sum of R^2 values from columns 2 to 5 will result in a R^2 value that does not equal, but rather exceeds, 1.

To determine whether the *LRDIMM* and *DDR4* variables have a statistically significant effect on our hedonic regression model, we also ran (1) a Wald test on only *LRDIMM* and (2) a Wald test on both *LRDIMM* and *DDR4* (to determine their joint significance). Table 3 presents the results of both Wald tests.

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[97] *See, e.g.*, Wooldridge, *supra* note 1, at 40 ("[T]he value of R^2 is always between zero and one.").

Table 3. Wald Test Results for Individual
Variables and Combined Variables

Variable(s)	*F*-Statistic	*p*-Value
LRDIMM	8.21	0.0047***
LRDIMM + DDR4	7.22	0.0010***

Notes: * indicates statistical significance at the 90-percent confidence level, ** indicates statistical significance at the 95-percent confidence level, and *** indicates statistical significance at the 99-percent confidence level.

For the Wald test on only *LRDIMM*, we obtained an *F*-statistic of 8.21 and a *p*-value of 0.0047. For the Wald test on both *LRDIMM* and *DDR4*, we obtained an *F*-statistic of 7.22 and a *p*-value of 0.0010. These results enable us to reject with 99 percent confidence the null hypothesis that the *LRDIMM* variable individually, or the null hypothesis that the *LRDIMM* and *DDR4* variables jointly, have no effect on our hedonic price model.

D.   *The Implementer's Incremental Cost of Producing an LRDIMM Rather Than an RDIMM*

Before apportioning the incremental value of the LRDIMM standard among the various contributors to that standard, one must first subtract from that incremental value the implementer's incremental cost of producing an LRDIMM rather than an RDIMM. As we have explained in Part IV.A, that incremental cost of production is attributable to the additional nine distributed data buffers (DBs) that an implementer attaches to the printed circuit board (PCB) of a DDR4 LRDIMM,[98] which Figure 4 shows.

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[98]   *See, e.g.*, Hewlett Packard Enterprise, DDR4 LRDIMMs Let You Have It All 3 (2016), https://www.hpe.com/h20195/v2/getpdf.aspx/4AA6-6067ENW.pdf?ver=1.0 (showing that "nine data buffers" are present on an LRDIMM). The nine distributed data buffers are not present on the PCB of an RDIMM. *See, e.g.*, Rambus, Server DIMM Chipsets 2 (2016), https://42xtjqm0qj0382ac91ye9exr-wpengine.netdna-ssl.com/wp-content/uploads/2016/08/Server-DIMM-Chipsets-Solution-Overview.pdf (comparing an LRDIMM with an RDIMM); *DDR4 Products*, Montage Technology, http://montage-tech.com/DDR4/index.html; Samsung, Module Handling Guide 3 (2015), http://www.samsung.com/semiconductor/global/file/productlanding/2015/08/Module_Handling_Guide-1.pdf.

Figure 4. Distributed Data Buffers on a DDR4 LRDIMM
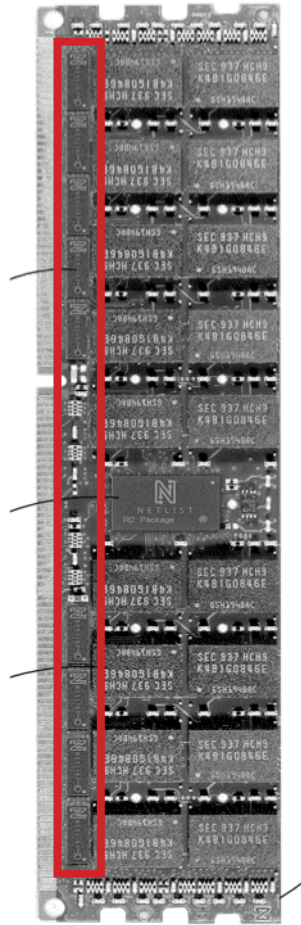
Source: U.S. Patent No. 8,516,185 fig. 3C (Apr. 15, 2010).
Note: The red box outlines the nine distributed data buffers. The image here is presented vertically, but the module is typically slotted horizontally in a server, 90 degrees counterclockwise from its presentation here.

We conservatively assume that the nine distributed data buffers add about $2.30 each to the production cost of a memory module relative to an RDIMM, for a modest production volume. A large-scale producer could potentially negotiate better prices from its suppliers and consequently capture more of the surplus. Thus, for simplicity of exposition, we assume that LRDIMM products cost, on average, $20.70 more per module to manufacture than an RDIMM product. Put differently, we assume that the incremental cost per unit when shifting production from RDIMMs to LRDIMMs is $20.70 per module.

We understand that the addition of the nine distributed data buffers on a DDR4 LRDIMM does not require a memory-module producer to install any additional manufacturing equipment. Thus, the manufacturer's cost of assembly would not materially change when shifting production from RDIMMs to LRDIMMs. Consequently, one can identify the implementer's incremental cost as solely the cost of the additional distributed data buffer components installed on the memory module's PCB, such that the net value to an implementer of the LRDIMM standard for an individual practicing product is $79.77 (that is, $100.47 − $20.70), as determined by what customers have demonstrated they are willing to pay.

Next, in Part V, we apportion that net incremental value of $79.77 to the patents essential to JEDEC's LRDIMM standard.

V. Using Forward-Citation Analysis to Apportion the Value That an SEP Holder's Portfolio Contributes to the Standard

Here, we report the results of the second step of our analysis—a forward-citation analysis to apportion the net incremental value of the LRDIMM standard across holders of patents essential to that standard. Some SEPs might contribute a greater share of value to a given standard relative to other SEPs for that standard, such that a small portion of patents essential to that standard might contribute the majority of the standard's value. This question of course is ultimately empirical, and its answer will vary from one standard to another.

Not to make this adjustment for the value distribution of SEPs would be an error of economic analysis. Economists have roundly criticized unweighted patent counting because it simplistically assumes that all patents in a portfolio have equal value.[99] That assumption is misleading because firms can file multiple patents for the same technology in different countries or declare more patents to be essential to a standard than are essential in fact.[100] In

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[99] *See, e.g.*, Anne Layne-Farrar, A. Jorge Padilla & Richard Schmalensee, *Pricing Patents for Licensing in Standard Setting Organizations: Making Sense of FRAND Commitments*, 74 Antitrust L.J. 671, 682–85 (2007); F.M. Scherer & Dietmar Harhoff, *Technology Policy for a World of Skew-Distributed Outcomes*, 29 Res. Pol'y 559 (2000); Mark Schankerman & Ariel Pakes, *Estimates of the Value of Patent Rights in European Countries During the Post-1950 Period*, 96 Econ. J. 1052 (1986); Sidak, *The Meaning of FRAND, Part I: Royalties*, *supra* note 91, at 1019–20, 1049–52; Jean O. Lanjouw, Ariel Pakes & Jonathan Putnam, *How to Count Patents and Value Intellectual Property: The Uses of Patent Renewal and Application Data*, 46 J. Indus. Econ. 405 (1998); Manuel Trajtenberg, *A Penny for Your Quotes: Patent Citations and the Value of Innovations*, 21 RAND J. Econ. 172 (1990); David J. Teece, Peter C. Grindley & Edward F. Sherry, SDO IP Policies in Dynamic Industries, Submission to the ITU Patent Roundtable 19 (Oct. 10, 2012) ("[T]here is no reason to believe that the value of different patents (or portfolios of patents) is proportional to the number of patents in the portfolio, even for 'essential' patents."); *see also In re* Innovatio IP Ventures, LLC Patent Litig., No. 11 C 9308, 2013 WL 5593609 (N.D. Ill. Oct. 3, 2013).

[100] *See, e.g.*, Sidak, *The Meaning of FRAND, Part I: Royalties*, *supra* note 91, at 1049.

Innovatio, Judge James Holderman recognized the unreliability of the naked patent-counting method, saying that "[i]f a patent holder owns ten out of a hundred patents essential to a given standard, it does not automatically mean that it contributes 10% of the value of the standard."[101] To be reliable and intellectually rigorous, an economic assessment of the value of an SEP portfolio must use a methodology that controls for the variation in value across the relevant universe of SEPs.[102]

A brief digression can help to illustrate the shortcomings of certain patent-ranking methodologies that expert economic witnesses have used in litigation. In *Innovatio*, economist Gregory Leonard opined on the value of Innovatio's patents using Mark Schankerman's well-known analysis,[103] which found that "the top 10% of all [non-standard-essential] electronics patents account for 84% of the value in all electronics patents."[104] Leonard, however, takes Schankerman's methodology out of context. For example, to determine the distribution of patent value among SEPs of the IEEE's 802.11 WiFi standard, Leonard relied on the findings of Schankerman's 1998 study, which in turn is based on patent-renewal data from 1970 to 1987 for all patent applications in France between 1969 and 1982.[105] Some key differences between SEPs and standard-*in*essential patents prevent reliably extrapolating to SEPs the results of Schankerman's 1998 study, which examined patents in general (regardless of whether they were essential to a standard).

Typically, when a patent holder declares its patent to be standard-essential, it must also commit to offer to license that patent on RAND or FRAND terms. In addition, one would expect renewal rates of SEPs to exceed the renewal rates of standard-inessential patents, which were the primary metric by which Schankerman measured patent value. Furthermore, Schankerman's 1998 study examined the value of patent protection, which does not necessarily equal the value of the royalty from licensing a patent. A patent grants the patent holder the right to exclude others from using the patented technology. The value of that right to the patent holder includes both the value of the competitive advantage from excluding one's competitors from using the patented technology and the value of the royalties from licensing the patent. Consequently, Schankerman's 1998 study, which examines the value of patent protection, estimates the sum of the value of the competitive advantage from

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[101]   *Innovatio*, 2013 WL 5593609, at *10.

[102]   *See* GPNE Corp. v. Apple, Inc., No. 12-cv-02885, 2014 WL 1494247, at *7 (N.D. Cal. Apr. 16, 2014) ("Patent counting, or counting the number of patents essential to a standard and determining the value of a single patent by dividing the value of the standard by the number of essential patents, is imprecise because it does not account for the value of the asserted patent relative to the other standard essential patents.").

[103]   Mark Schankerman, *How Valuable Is Patent Protection? Estimates by Technology Field*, 29 RAND J. ECON. 77, 80 (1998).

[104]   *Innovatio*, 2013 WL 5593609, at *43.

[105]   Schankerman, *supra* note 103, at 80.

excluding other users *and* the value of royalties from licensing. However, because SEPs—in contrast to ordinary patents—are subject to a RAND or FRAND commitment, most, if not all, of the value of SEPs consists of the value of the royalties from licensing. Consequently, extrapolating the results of Schankerman's 1998 study of non-SEPs to estimate the value of present-day SEPs would be speculative and unreliable.

To account for the possibility that some SEPs contribute relatively greater value to the standard than do others, we use forward-citation analysis of patents essential to JEDEC's LRDIMM standard to derive the value distribution of those SEPs. The starting point for our patent-ranking methodology is an examination of each individual licensing assurance (also known as a letter of assurance (LOA) in the argot of standard setting). The license assurance is the declaration to JEDEC by a patent holder that the identified patent is potentially essential to a specific JEDEC standard. A specimen of a JEDEC license assurance appears in Appendix I. Using data from the submitted licensing assurances to JEDEC, we first identify the universe of active U.S. patents that patent holders have declared to be essential to JEDEC's LRDIMM standards.

We then apportion the relative value that Netlist's LRDIMM SEP portfolio contributes to the total value of the LRDIMM standard. To apportion that value, we calculate the relative value that Netlist's SEPs contribute to the LRDIMM standard using a weighted patent-citation score for each SEP. That is, as we explain below, we weight the value of each SEP by its forward citations to account for that SEP's value, relative to other SEPs, for the LRDIMM standard. We adjust the weighted value of each SEP using both a patent-citation score that includes self-citations (that is, forward citations that a patent receives from a newly issued patent owned by the same company) as well as a patent-citation score that excludes self-citations.

Our empirical analysis explained below indicates that customers demonstrate that they are willing to pay $34.27 per memory module for the added benefits flowing from Netlist's LRDIMM SEP portfolio.

A. Identifying Patents Declared Essential to JEDEC's LRDIMM Standard

To determine the value that Netlist's LRDIMM SEP portfolio contributes to the total value of the LRDIMM standard, we first analyzed data on license assurances that patent holders submitted to JEDEC disclosing their patents as potentially essential to that standard. To identify patents essential to the LRDIMM standard, we reviewed JEDEC's database of patents declared essential to the LRDIMM standard. Then, using each SEP holder's submitted keywords describing the relevant technology in that database, we created a list of keywords with which to search JEDEC's database of

submitted license assurances. The keywords that we used for identifying license assurances that disclosed patents essential to the DDR4 LRDIMM and RDIMM standards were Load decoupling DIMM, LD-DIMM, LD DIMM, LDDIMM, LR-DIMM, LR DIMM, LRDIMM, Load reducing DIMM, Reduced load DIMM, Reduced load DIMMS, RDIMM Keywords, RDIMM, and Registered DIMMs. Capitalization and hyphenation do not affect the results of this keyword identification.

This procedure identified license assurances that had declared patents as being essential to the LRDIMM standard, such as the one by Netlist that we include as an example in Appendix I. We then inspected summaries of each license assurance, which JEDEC maintains, to ensure that our search successfully identified the relevant license assurances and that the terms that we used related to the terms that SEP holders themselves used to describe the technology covered by their declared essential patents. If a patent were erroneously excluded from this analysis, the exclusion would need to be systematically biased for or against a particular SEP holder submitting license assurances to bias our apportionment results. Otherwise, the omission would be harmless error.

We then compiled a list of those declared SEPs that enabled us to identify a master list of 50 active U.S. patents essential to JEDEC's LRDIMM standard. We further verified those SEPs with publicly available data from the PatentsView database compiled by the U.S. Patent and Trademark Office (USPTO) and identified additional data regarding those SEPs.[106] The USPTO data include variables for an SEP's features, claims, grant and expiration dates, and forward citations (which measure when the patent of interest is cited as relevant for a later patent, either by the later patent's applicant or by a patent examiner).

We excluded expired patents from our primary analysis. Expired patents are in the public domain. If these patents were relevant to the prior standard (in this case, RDIMM), then their value will be measured by the constant term in our hedonic regression model. Otherwise, their value might be captured by consumers in the form of lower prices or by the manufacturer in the form of higher profit margins or lower prices, or by both consumers and manufacturers in some combination. One cannot simply assume that the residual value of expired patents accrues exclusively to SEP owners. In Part V.F, we rerun our analysis with expired patents included to provide a

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[106] *PatentsView*, UNITED STATES PATENT & TRADEMARK OFFICE [USPTO], http://www.patentsview.org/querydev/. PatentsView is "a patent data visualization and analysis platform intended to increase the value, utility, and transparency of prototype US patent data," an initiative supported by the USPTO's Office of Chief Economist. *FAQs-What Is PatentsView?*, USPTO, http://www.patentsview.org/api/faqs.html. The data are "sourced from USPTO-provided text and XML data on published patent applications (2001–present) and granted patents (1976–present)." *Id.*

conservative estimate of the relative value of the LRDIMM standard that is attributable to new public-domain technologies.

We also excluded patent applications from our analysis. A patent application does not represent a valid intellectual property right for use in our hedonic regression model. It presents additional data issues, as an application might later be granted and included in the standard as an identified SEP or abandoned during the application process. Including patent applications consequently would create a risk of double counting a single technology or counting a technology that is in fact not patentable. Moreover, there is no *a priori* reason to believe that including patent applications in the analysis would (1) change the distribution of patent value for the LRDIMM standard (such that the rankings of companies by patent value would change) or (2) otherwise systematically bias the ranking. Thus, absent any compelling reason to believe that the effect of including patent applications would be so weighty as to dominate the effect of granted patents alone in our ranking methodology, we intentionally exclude patent applications from our analysis. Similarly, standards may be revised from time to time, with new technologies being added or older technologies being dropped. However, we do not believe that this process occurs in a way that can be predicted before the establishment of a standard. Thus, we model the apportionment of patent value as a static (rather than a dynamic) system, where the likeliest outcome of proportional value after any revision is the currently observable distribution.

B.   *Using Forward Citations to Create a Score to Measure a Patent's Relative Value*

We assessed the value of Netlist's patents essential to the LRDIMM standard relative to the value of other SEP holders' patents essential to the LRDIMM standard using data from the PatentsView database, compiled by the U.S. Patent and Trademark Office (USPTO), on the number of forward citations that each SEP in the LRDIMM standard received over time.[107] A forward-citation count is the number of citations that an issued patent receives from subsequently issued patents. Economists frequently use a patent's forward citations as a proxy measure of that patent's value.[108]

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[107]   *PatentsView*, United States Patent & Trademark Office, http://www.patentsview.org/querydev/.

[108]   *See, e.g.*, Manuel Trajtenberg, *A Penny for Your Quotes: Patent Citations and the Value of Innovations*, 21 RAND J. Econ. 172 (1990); Adam B. Jaffe, Manuel Trajtenberg & Rebecca Henderson, *Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations*, 108 Q.J. Econ. 577 (1993); Dietmar Harhoff, Francis Narin, F.M. Scherer & Katrin Vopel, *Citation Frequency and the Value of Patented Inventions*, 81 Rev. Econ. & Stats. 511 (1999); Bhaven N. Sampat & Arvids A. Ziedonis, *Patent Citations and the Economic Value of Patents: A Preliminary Assessment*, *in* Handbook of Qualitative Science and Technology Research 277 (Springer 2005); Adam B. Jaffe, Manuel Trajtenberg & Michael S. Fogarty, *The Meaning of Patent Citations: Report on the NBER/Case-Western Reserve Survey of Patentees*, *in* Adam B. Jaffe & Manuel Trajtenberg, Patents, Citations & Innovations 379 (MIT Press 2002). Forward-citation analysis is a subset of bibliometrics. *See, e.g.*, Nicola de Bellis, Bibliometrics and Citation Analysis: From the Science Citation Index to Cybermetrics (Scarecrow Press 2009); Beyond Bibliometrics: Harnessing

Furthermore, multiple U.S. courts have recognized that forward-citation analysis, when sufficiently tied to the facts of the case, is a reliable and useful apportionment methodology to determine the value of a given patent relative to the value of other patents.[109] The rationale is that a patent referenced frequently by later inventions is likely to have made a greater relative contribution to knowledge and is therefore more valuable than a patent with fewer forward citations (assuming that the patents are the same age).

Nonetheless, there is an implicit bias in such forward-citation data: a patent that has existed for a longer time than other patents will tend to generate more forward citations merely due to the fact that other inventors have had more time to identify and cite that particular patent.[110] In Figure 5, we demonstrate that implicit bias by plotting the number of forward citations against the year in which the patent was granted for each patent in our estimation sample.


MULTIDIMENSIONAL INDICATORS OF SCHOLARLY IMPACT (Blaise Cronin & Cassidy R. Sugimoto eds., MIT Press 2014); YVES GINGRAS, BIBLIOMETRICS AND RESEARCH EVALUATION: USES AND ABUSES (MIT Press 2016); HENK F. MOED, CITATION ANALYSIS IN RESEARCH EVALUATION (Springer 2005); ROBERTO TODESCHINI & ALBERTO BACCINI, HANDBOOK OF BIBLIOMETRIC INDICATORS: QUANTITATIVE TOOLS FOR STUDYING AND EVALUATING RESEARCH (Wiley 2016).

[109] *See, e.g.*, Intel Corp. v. Future Link Sys., LLC, No. 14-cv-00377, 2017 WL 2482881, at *5 (D. Del. June 1, 2017) ("In cases where forward citation analysis has been found unreliable, it was because the expert failed to 'tie the methodology to the facts' . . . ." (quoting Finjan, Inc. v. Blue Coat Sys., Inc., No. 13-cv-03999, 2015 WL 4272870, at *8 (N.D. Cal. July 14, 2015)); Comcast Cable Commc'ns, LLC v. Sprint Commc'ns Co., 218 F. Supp. 3d 375, 383 (E.D. Pa. 2016) ("[C]ourts have not rejected forward citation analysis outright."); Better Mouse Co. v. SteelSeries ApS, No. 2:14-cv-00198, 2016 WL 3611528, at *2 (E.D. Tex. Jan. 5, 2016) ("To the extent Plaintiff claims that forward citation analysis is never relevant for patent valuation, the Court rejects that claim. No binding authority states that forward citation analysis is per se not relevant to the facts of any case."); *Finjan v. Blue Coat*, 2015 WL 4272870, at *7 ("[A] qualitative analysis of asserted patents based upon forward citations may be probative of a reasonable royalty in some instances."); Comcast Cable Commc'ns, LLC v. Sprint Commc'ns Co., No. 2-12-cv-00859, slip op. at 41 (E.D. Pa. Aug. 16, 2017) ("[F]orward citation analysis is reliable when . . . it is tied to the facts of the case.").

[110] *See* Trajtenberg, *A Penny for Your Quotes: Patent Citations and the Value of Innovations*, *supra* note 108, at 186; *see also Finjan v. Blue Coat*, 2015 WL 4272870, at *8 (criticizing the expert economic witness's forward-citation analysis for failing to account for the age bias in the forward-citation data).

Figure 5. Relationship Between the Number
of Forward Citations and Patent-Grant Year
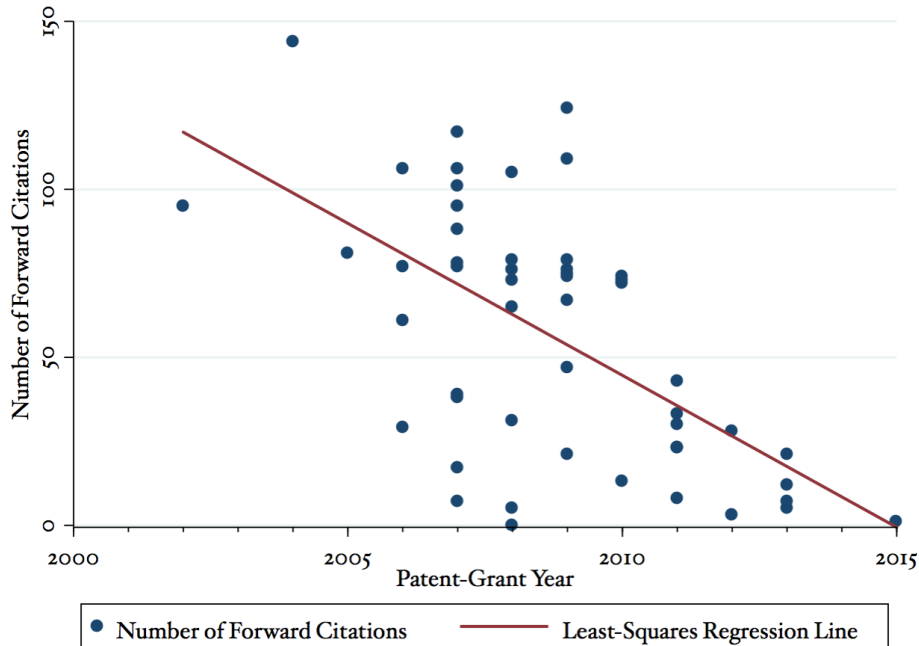
Source: *PatentsView*, USPTO, http://www.patentsview.org/query/.
Note: Authors' analysis. This figure is exported from the Stata statistical software.

In Figure 5, the least-squares regression line—that is, the line of best fit to the scatterplot—has a slope of −9.04, indicating a negative relationship between the patent-grant year and the number of forward citations. Put differently, each additional year in a patent's age is associated with approximately nine more forward citations to that patent. That relationship is associated with a *p*-value of 0.000 and is thus statistically significant at the 99-percent confidence level. To control for this bias, we rescale each patent's citation score so that later citations weigh less than the earlier citations that a patent receives. A patent that receives 10 citations within a year of its issue will thus be counted as more valuable than a patent that receives 10 citations within 10 years of issuance. We express this relationship for an individual patent in the following equation:

$$\textit{Patent-Citation Score} = \sum_{\text{Citations}} \left(\frac{1}{2}\right)^{t/h}, \qquad (5)$$

where *t* is the number of days between the date of each cited patent's issuance and the date of the citing patent's issuance, and *h* is the half-life of a patent citation—that is, the median number of days that elapse between forward

citations for all patents analyzed. Using the median number of days until a citation as the half-life means that a citation that occurs at that time will have a score of 0.5. The half of citations that occur closer to the patent grant date will have scores closer to one, and the other half of citations, which are made a longer time after the grant date, will have scores closer to zero.

For example, suppose patent A receives three citations within a month—one citation from a patent that has been issued 10 days since the issuance of patent A, one citation from a patent that has been issued 20 days since the issuance of patent A, and one citation from a patent that has been issued 30 days since the issuance of patent A. The citation from the patent issued 10 days after the initial grant has a score above 0.5, the citation at 20 days has a score of 0.5, and the citation at 30 days has a score below 0.5. The total patent-citation score for patent A is the sum of these three individual scores,

$$\left(\frac{1}{2}\right)^{10/20} + \left(\frac{1}{2}\right)^{20/20} + \left(\frac{1}{2}\right)^{30/20} = 1.56. \tag{6}$$

That is, the total patent-citation score for patent A is 0.71 + 0.50 + 0.35 = 1.56.

It also bears emphasis that some district courts have found forward-citation analysis to be an unreliable methodology for analyzing the value of a reissued patent when the expert fails to account for forward citations to the reissued patent's predecessors. A reissued patent is an issued patent that was found to be defective, and thus invalid, but was subsequently reissued upon correction of the defect.[111] In *Oracle America, Inc. v. Google, Inc.*, the U.S. District Court for the Northern District of California excluded Gregory Leonard's testimony regarding a reissued patent's relative value because he did not count forward citations to the reissued patent's predecessors.[112] Nonetheless, in *Better Mouse Co. v. SteelSeries ApS*, the U.S. District Court for the Eastern District of Texas distinguished this evidentiary ruling in *Oracle* by explaining that "the plaintiff in *Oracle* made a showing, not made here, that the difference in outcomes was so significant as to render the analysis unreliable on those facts."[113] An expert can determine whether a significant change in the outcome arises by examining both scenarios to determine which method is more appropriate given the facts of the case. There are no reissued patents included in our analysis.

Table A.1 in Appendix III lists the 50 active U.S. patents that we use in our forward-citation analysis, ranked by their patent-citation scores (as we

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[111] 35 U.S.C. § 251; *see* Kenie Ho, Esther H. Lim & Carles E. Van Horn, *Effective Uses of Reissues and Reexaminations in the United States*, CHINA IP NEWS (June 2009), http://www.finnegan.com/resources/articles/articlesdetail.aspx?news=6424bc6c-0369-48a8-963d-014909818582.

[112] No. 10-cv-03561, 2012 WL 877125, at *2 (N.D. Cal. Mar. 15, 2012) (excluding Leonard's testimony on the partial basis that he "fail[ed] to account for the fact that [one of the patents in suit] was re-issued twice, and thus fail[ed] to include citation counts to its predecessor patents").

[113] No. 2:14-cv-00198, 2016 WL 3611528, at *2 n.1 (E.D. Tex. Jan. 5, 2016).

have computed it in accordance with the weighting procedure explained in Equation 5 above). In addition to reporting each patent's patent-citation score, Table A.1 reports each patent's number, current assignee, title, issue date, and number of forward citations.

C.   *The Value Distribution of Patents Declared Essential to the LRDIMM Standard*

Using the data on forward citations, we ordinally rank the 50 SEPs included in the LRDIMM standard according to their weighted citations. Graphing the cumulative value of the ranked SEPs produces a Lorenz curve,[114] a tool that economists have long used for measuring the inequality in distributions, such as the distribution of income within and among nations.[115] In recent years, scholars and expert economic witnesses in patent-infringement disputes have used the Lorenz curve as a method to rank and value patents or portfolios of patents.[116] Figure 6 shows the Lorenz curve for the patents declared essential to JEDEC's LRDIMM standard.

[114] For the definition of the Lorenz curve, see N.C. Kakwani, *Applications of Lorenz Curves in Economic Analysis*, 45 Econometrica 719, 719 (1977) ("The Lorenz curve relates the cumulative proportion of income units to the cumulative proportion of income received when units are arranged in ascending order of their income."); Daniel B. Levine & Neil M. Singer, *The Mathematical Relation Between the Income Density Function and the Measurement of Income Inequality*, 38 Econometrica 324, 324 (1970) ("The Lorenz curve exhibits income distribution by plotting the interdependence of . . . the percentage of total income earned by the percentage of population.").

[115] *See, e.g.*, Kakwani, *supra* note 114, at 719; Martin Bronfenbrenner, Income Distribution Theory 47–50 (Aldine Transaction 1971).

[116] *See* Sidak, *The Meaning of FRAND, Part I: Royalties*, *supra* note 91, at 1035–36; Jonathan D. Putnam, *Value Shares of Technologically Complex Products* 15 (Competition Dynamics Working Paper, Apr. 16, 2014), http://www.competitiondynamics.com/wp-content/uploads/Value-Shares-20140416.pdf.

Figure 6. Lorenz Curve Depicting the Distribution of
Value of Patents Declared Essential to JEDEC's
LRDIMM Standard, Ranked by Forward Citations
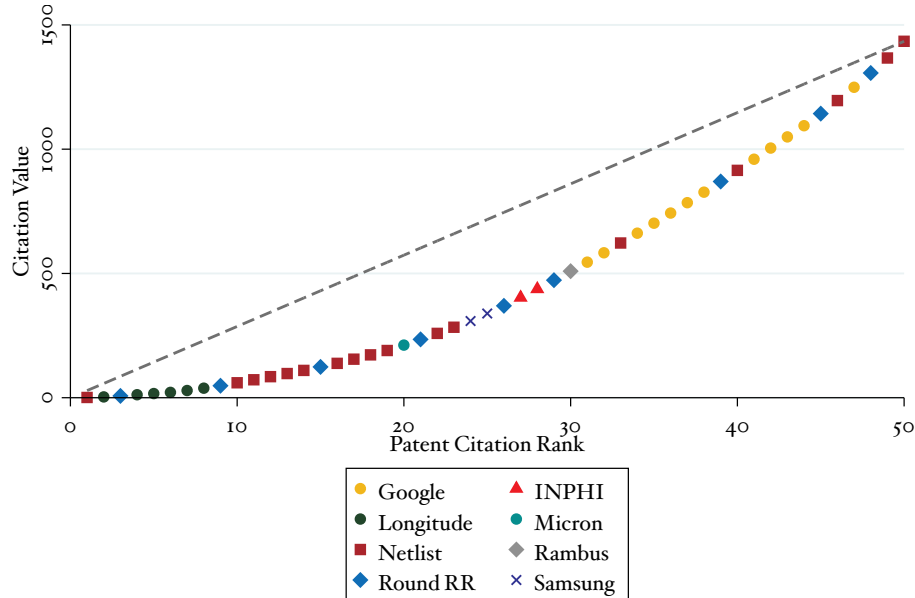
Source: *PatentsView*, USPTO, http://www.patentsview.org/query/.
Note: Authors' analysis. This figure is exported from the Stata statistical software.

In Figure 6, the horizontal axis represents the ranked value of each LRDIMM SEP. The higher the SEP's forward-citation rank, the higher the ranked value of the SEP. The patent at the farthest left of the graph has the lowest forward-citation rank, and the patent at the farthest right of the graph has the highest forward-citation rank. Each SEP's "height" in the graph—that is, its value on the vertical axis—is the cumulative scored value of its forward citations. The straight line in Figure 6 is the line that would be drawn if each SEP's citation score were the same. That is, if all LRDIMM SEPs were equally valuable (as measured by forward citations), all the points on the graph would fall along the straight line depicted. That outcome could occur if, for example, each SEP had an equal number of forward citations.

The convex shape of the dotted curve in Figure 6 indicates that the value of the standard is not distributed equally across the patents declared to be essential to the LRDIMM standard. The degree to which the Lorenz curve deviates from the 45-degree line (represented by the diagonal gray line in Figure 6) is a measure of the skewness of the distribution of value of the patents declared to be essential to the LRDIMM standard and thus a measure of the inequality of the value that those SEPs contribute to the standard. The dotted curve reveals, for example, that a given SEP that has

a patent-citation rank of 10 contributes relatively less value to the standard than an SEP that has a patent-citation rank of 40. As one proceeds up the patent-citation rank, the value of each SEP is added to the cumulative value of the SEPs ranked below it. In other words, the change in the cumulative value of the LRDIMM standard upon adding one more SEP to the standard is equal to that SEP's individual value.[117]

D. *Apportioning the Value of the LRDIMM Standard on the Basis of Forward Citations, Including Self-Citations*

Using the derived Lorenz curve in Figure 6, we determine each company's contribution to the LRDIMM standard by summing the weighted citation value of each patent across all companies holding SEPs for that standard. Figure 7 shows the distribution of patent value by company in JEDEC's LRDIMM standard based on the number of forward citations.

Figure 7. Distribution of Value by Company
of LRDIMM SEPs Derived from Forward Citations
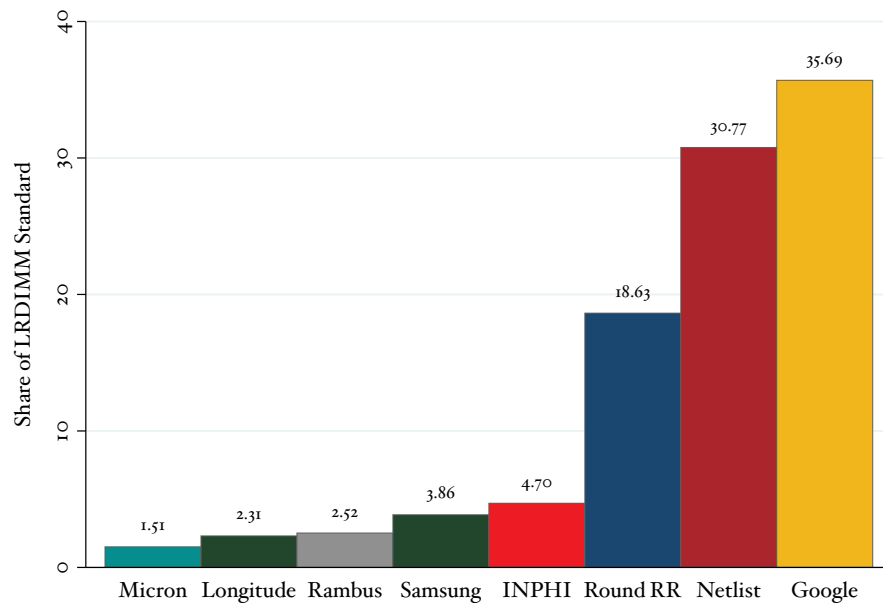
Source: *PatentsView*, USPTO, http://www.patentsview.org/querydev/.
Note: Using data from PatentsView, we weighted by forward citations the distribution of LRDIMM SEPs across SEP holders.

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[117] In contrast to our numerical estimation of the Lorenz curve (explained in greater detail below and in Appendix II), Putnam's methodology, *supra* note 116, employs a differentiable function that enables him to evaluate the incremental contribution of any given patent (relative to the contribution of the "average" patent situated at the 50th percentile of the ranking) by evaluating the slope of the Lorenz curve at the percentile of the patent in question (by evaluating the derivative of the Lorenz curve at that point).

As Figure 7 shows, this forward-citation analysis indicates that Netlist's LRDIMM SEPs account for 30.77 percent of the value of the technology included in the LRDIMM standard. Consequently, the value of JEDEC's LRDIMM standard that is attributable to Netlist's LRDIMM SEPs is $79.77 × 30.77 percent = $24.55.

E. *Apportioning the Value of the LRDIMM Standard on the Basis of Forward Citations, Excluding Self-Citations*

One critique of counting forward citations to derive a patent's value relative to other patents is that a patent holder might extravagantly cite its own (older) patent in its newly issued patents—a practice called "self-citation." Thus, self-citations might exaggerate the older patent's value.[118] To evaluate the effect of self-citations on our patent-ranking analysis of the LRDIMM standard, we identify the assignee of each citing patent. If the assignee of the *citing* patent is the same as the assignee of the *cited* patent, we count that forward citation as a self-citation. We assign each self-citation a value of zero and then recalculate each LRDIMM SEP's citation score.

Figure 8 below compares, for each LRDIMM SEP, the SEP's weighted citation score *including* self-citations (the citation value on the *y*-axis) with that same SEP's weighted citation score *excluding* self-citations (the citation value on the *x*-axis). An LRDIMM SEP that has a high citation score on the *y*-axis but a low citation score on the *x*-axis has a higher proportion of self-citations than an SEP that has a high citation score on both the *y*-axis and the *x*-axis. If an SEP's citation score on the *y*-axis equals its citation score on the *x*-axis (that is, if the citation score lies along the 45-degree line shown), then that SEP has no self-citations, which implies that all of the citation value for that particular SEP comes from other companies' patent citations.

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[118] At least one court has excluded an expert witness's forward-citation analysis, in part, for failing to account for self-citations. In *Finjan, Inc. v. Blue Coat Systems, Inc.*, Judge Beth Freeman of the Northern District of California observed that "a patent's objective quality cannot be based on the number of times an inventor cites himself in prosecuting related patents." No. 13-cv-03999, 2015 WL 4272870, at *8 (N.D. Cal. July 14, 2015) (excluding the expert testimony of Anne Layne-Farrar).

Figure 8. Citation Score of Patents Declared Essential
to JEDEC's LRDIMM Standard Ranked by Forward
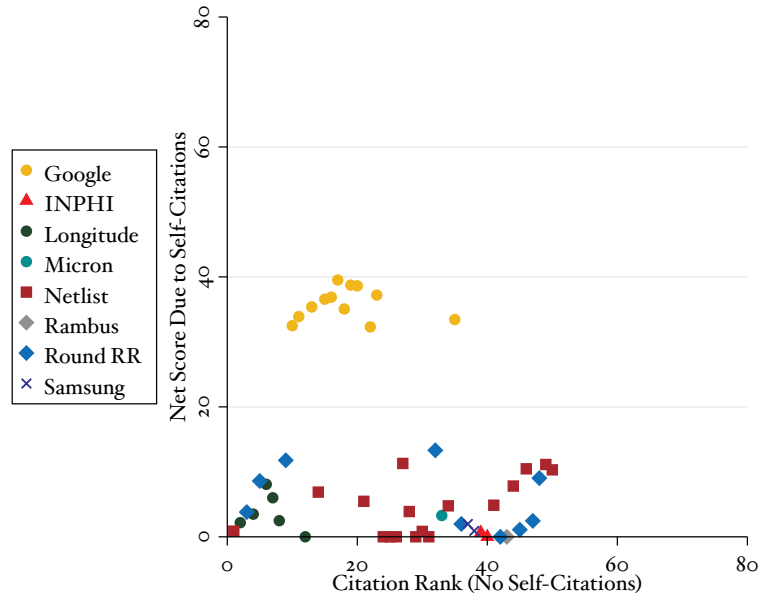Citations, Both with and without Self-Citations
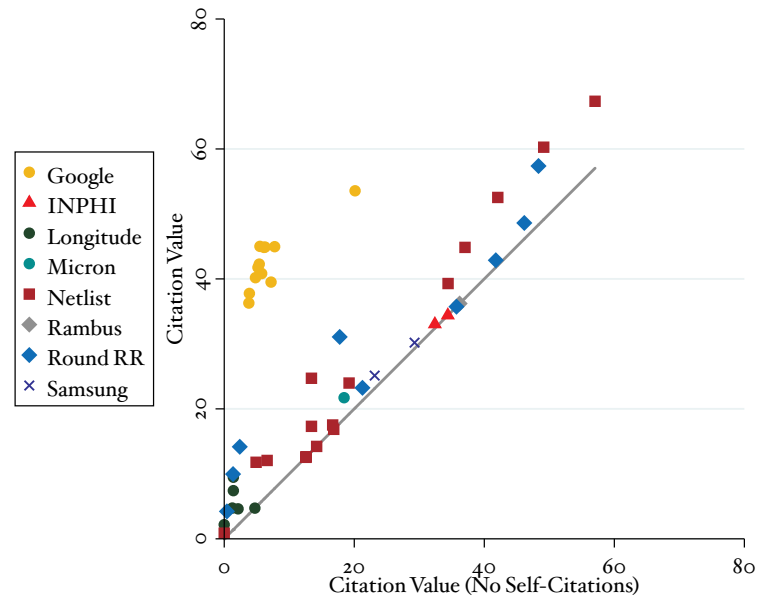
Source: *PatentsView*, USPTO, http://www.patentsview.org/query/.
Note: Authors' analysis. This figure is exported from the Stata statistical software.

In Figure 8, the vertical distance from the data point for a single LRDIMM
SEP to the 45-degree line measures the amount of self-citations for that SEP.
Data points on the 45-degree line have no self-citations. Figure 8 reveals
that several SEPs have higher values when self-citations are included in the
patent-citation score than when self-citations are excluded, because the SEP
holder in question owned many of the later patents citing its earlier SEP.
However, the patent-citation score of most SEPs, except for the group of
SEPs currently owned by Google, declines by about the same amount across
all relevant SEPs when excluding the self-citations, which accounts for the
relatively linear distribution displayed in Figure 8.

Figure 9 below shows the same information, but with the *y*-axis shifted to
show only the net value of the patent score that is attributable to self-citations.

Figure 9. Net Citation Score of Patents Declared
Essential to JEDEC's LRDIMM Standard, Ranked by
Forward Citations, That Is Attributable to Self-Citations

Figure 9 reveals that several patents declared essential to JEDEC's LRDIMM standard have large citation scores attributable to self-citations. Figure 10 below shows, as a portion of each patent's total score, what portion is attributable to self-citations. Put differently, Figure 10 shows the proportion of each patent's score that is attributable to self-citations, rather than the actual score itself.

Figure 10. Proportion of Each Patent's Citation Score
of Patents Declared Essential to JEDEC's LRDIMM
Standard That Is Attributable to Self-Citations

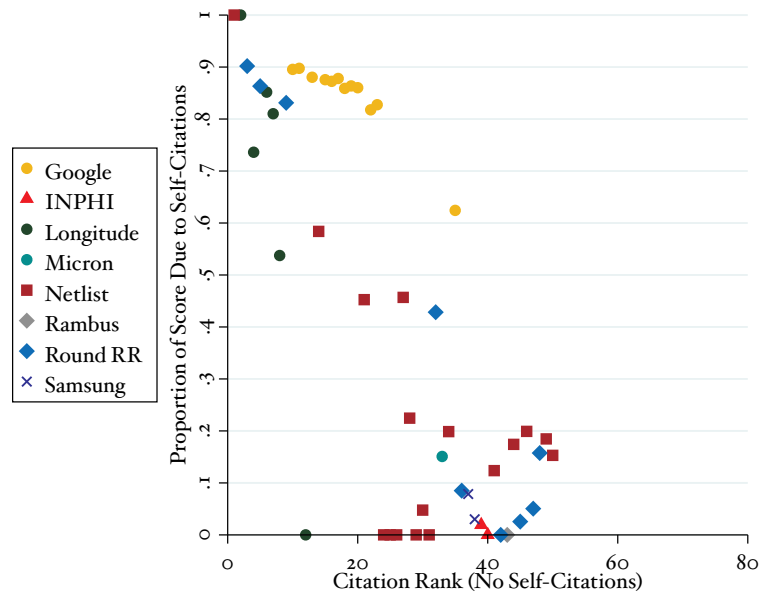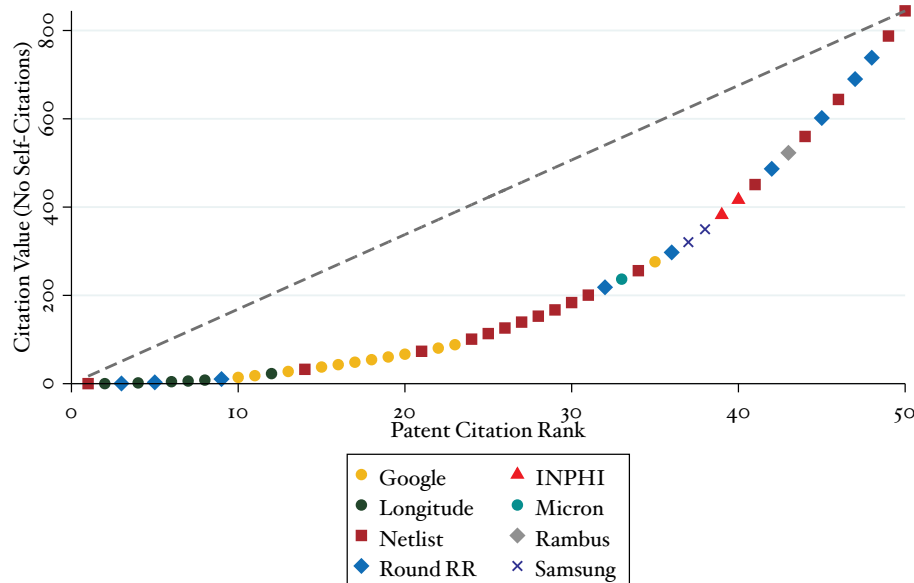*Source*: *PatentsView*, USPTO, http://www.patentsview.org/query/.
Note: Authors' analysis. This figure is exported from the Stata statistical software.

Figure 10 reveals that, for some LRDIMM SEPs, a large proportion of the citation score is attributable to self-citations. For example, for all but one of Google's LRDIMM SEPs, self-citations account for between 80 percent and 90 percent of the citation score. If one were to exclude self-citations to recalculate each company's contribution to JEDEC's LRDIMM standard, a company that attributes a higher proportion of its value to self-citations will experience a relatively larger decline in value (and therefore a larger decline in its perceived contribution to the LRDIMM standard) relative to a company that attributes a lower proportion of its value to self-citations. We rescale each company's patent-citation scores by excluding self-citations and then recalculate each SEP's rank and relative contribution to the LRDIMM standard. Figure 11 below presents the revised Lorenz curve for the patents declared essential to JEDEC's LRDIMM standard.

Figure 11. Lorenz Curve Depicting the Value Distribution
of Patents Declared Essential to JEDEC's LRDIMM Standard,
Ranked by Forward Citations, Excluding Self-Citations

Source: *PatentsView*, USPTO, http://www.patentsview.org/query/.
Note: Authors' analysis. This figure is exported from the Stata statistical software.

Again, the convex shape of the dotted curve in Figure 11 indicates that the value of JEDEC's LRDIMM standard is not distributed equally across the declared-essential patents. We have ranked the LRDIMM SEPs according to their new citation scores. To compare this graph to the graph in Figure 6 (which includes self-citations), we calculate the Gini coefficient for each graph. The Gini coefficient represents the ratio of the area between the Lorenz curve and the 45-degree line to the entire area under the 45-degree line and is "the most common measure of inequality."[119] In a distribution in which each patent has an equal citation score, each patent would lie on the 45-degree line, and the Lorenz curve would have a Gini coefficient of zero.[120] In contrast, a standard whose value is attributable solely to one

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[119] Joseph L. Gastwirth, *The Estimation of the Lorenz Curve and Gini Index*, 54 Rev. Econ. & Stat. 306, 307 (1972); *see also* James Morgan, *The Anatomy of Income Distribution*, 44 Rev. Econ. & Stat. 270, 270 (1962) ("It has generally been agreed, after much discussion, that the best single measure of inequality is the proportion of the triangular area on a Lorenz diagram which falls between the Lorenz curve and the diagonal, often called the Gini Index of concentration." (citing Mary Jean Bowman, *A Graphical Analysis of Personal Income Distribution in the United States*, 35 Am. Econ. Rev., Sept. 1945, at 607, 608–28)); Laura Langer, *Measuring Income Distribution Across Space and Time in the American States*, 80 Soc. Sci. Q. 55, 57–58 (1999).

[120] *See, e.g.*, Morgan, *supra* note 119, at 281; Langer, *supra* note 119, at 58.

patent—a distribution that is "perfectly unequal"—would have a Gini coefficient of one. Thus, a higher Gini coefficient indicates higher inequality within a distribution. For JEDEC's LRDIMM standard, we find that the Lorenz curve including self-citations has a Gini coefficient of 0.34 and that the Lorenz curve excluding self-citations has a Gini coefficient of 0.49. We present our methodology for calculating the Gini coefficient in Appendix II. These results confirm that the inclusion of self-citations in the patent-citation methodology increases the apparent "equality" of valuations of patents declared essential to JEDEC's LRDIMM standard.

We also examine the "90/10 ratio," which is the value of the 90th percentile of the distribution divided by the 10th percentile of the distribution. This statistic tests for inequality at the extremes of a distribution. A higher value indicates greater inequality. Our patent ranking for the LRDIMM standard that includes self-citations has a 90/10 ratio of 61.74.[121] In contrast, our patent ranking for the LRDIMM standard that excludes self-citations has a 90/10 ratio of 167.14.[122] Again, we find that including self-citations increases the apparent equality among patent values for the LRDIMM standard and assigns more of the standard's value to lower-ranked patents. One possible explanation for this finding is that patents that are cited less by other companies have a higher proportion of self-citations. Testing that hypothesis transcends the scope of this article.

Using the weighted citation values excluding self-citations, we reestimate each company's share of the value of JEDEC's LRDIMM standard. We show each company's recalculated share of the LRDIMM standard in Figure 12 below.

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[121] The patent at the 90th percentile of the distribution has a citation score of 1169.263 and the patent at the 10th percentile of the distribution has a citation score of 18.940. Thus, the 90/10 ratio is 1169.263 ÷ 18.940 ≈ 61.74. In other words, the patent at the 90th percentile of the distribution has a citation score almost 62 times as high as the citation score of the patent at the 10th percentile of the distribution.

[122] That is, 622.750 ÷ 3.720 ≈ 167.14. *See id.*

Figure 12. Distribution of Value by Company of LRDIMM SEPs
Derived from Forward Citations, Excluding Self-Citations
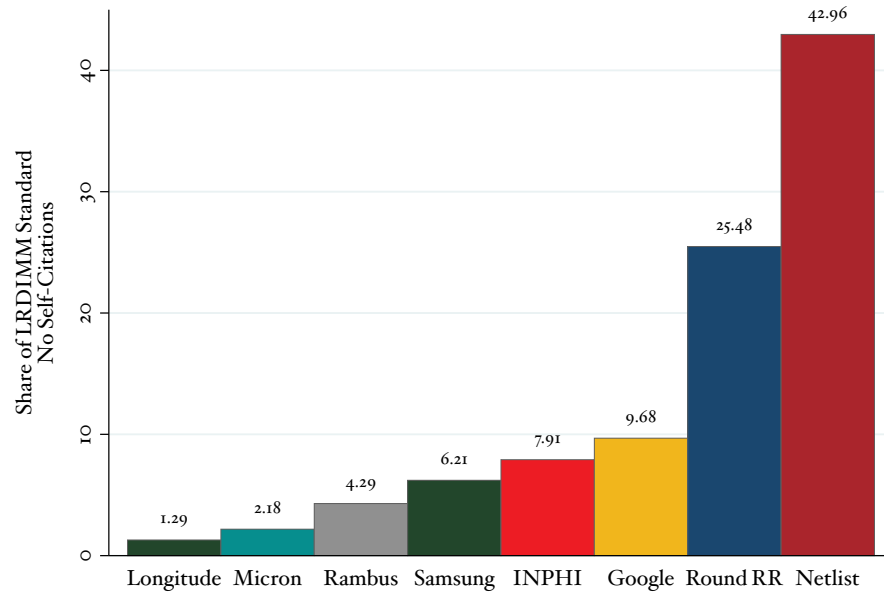
Source: *PatentsView*, USPTO, http://www.patentsview.org/querydev/.
Note: Using data from PatentsView, we weighted by forward citations the distribution
of DDR4 LRDIMM SEPs across SEP holders after removing self-citations.

As Figure 12 shows, a forward-citation analysis that accounts for self-citations indicates that Netlist's SEPs account for 42.96 percent of the value of the technology included in JEDEC's LRDIMM standard. The value of the LRDIMM standard attributable to Netlist's LRDIMM SEP portfolio is therefore \$79.77 × 42.96 percent = \$34.27.

F. Apportioning the Value of JEDEC's LRDIMM Standard Attributable to Technology in the Public Domain

We account for the value of the standard that is attributable to technology in the public domain by examining the value that our citation ranking assigns to expired patents that a firm had previously declared to be essential to JEDEC's LRDIMM standard. In our apportionment analysis described above, we excluded expired patents. In this part, we examine all granted patents (both active and expired) that have been declared essential to the LRDIMM standard. We apportion the value of active patents to the company that currently owns the patent, and we apportion the value of expired patents to a notional firm we call "Public."

This method ensures that we are accounting for value attributable only to *new* technologies in the public domain that were not already embodied in the alternative, next-best standard. Hence, we focus on analyzing technology essential to the LRDIMM standard that has only recently passed into the public domain (after the expiration of the patent protecting the technology). No firm may claim the value of that technology any longer; consequently, the technology is freely available for any implementer practicing the LRDIMM standard to use.

It bears emphasis that this method assumes that the value from expired patents should accrue to the implementer of the standard, rather than to the remaining SEP owners or to consumers. However, that assumption might not fit the relevant facts of a particular case. For example, in a given case, there might be evidence that consumers already receive the benefits of the standardized technology in the public domain through lower prices for standard-compliant products. In that scenario, our hedonic price model would implicitly account for the value of technology in the public domain—because the estimation of the standard's value relies on observed market prices for standard-compliant products—and an expert would not need to make any further adjustment to account for that value. Nonetheless, we perform the analysis here to show one approach to account for the value of standardized technology in the public domain.

Patents expire for multiple reasons, including the expiration of the patent's statutory term and the patent holder's failure to pay the patent's maintenance fees. Because some patents expire simply because of the passage of time, these patents might, on average, be older than active patents. Consequently, expired patents might have had more opportunity to be cited by later patents, which would inflate the citation score of expired patents relative to active patents. This feature of expired patents makes it essential to rescale each patent's citation score so that citations from later points in time weigh less than earlier citations that the patent receives, as we explained in Part V.B.

We find only three expired patents that were declared essential to JEDEC's LRDIMM standard. Figure 13 shows the distribution of patent value for all granted patents (including expired patents) declared essential to the LRDIMM standard according to each patent's citation score (including self-citations).

Figure 13. Distribution of Value by Company of All
Granted LRDIMM SEPs Derived from Forward
Citations, Including Self-Citations
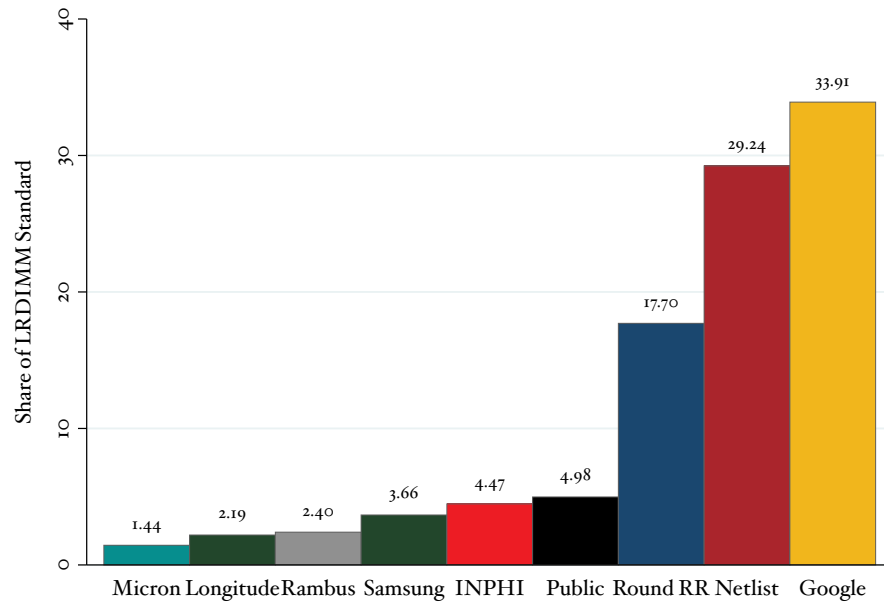
Source: *PatentsView*, USPTO, http://www.patentsview.org/querydev/.
Note: Using data from PatentsView, we weighted by forward citations the distribution of
DDR4 LRDIMM SEPs across SEP holders after removing self-citations. "Public" denotes
the totality of expired LRIDMM SEPs.

Figure 13 shows that the expired patents in the public domain account for
4.98 percent of the value of the LRDIMM standard. The three expired
patents contribute more value to the LRDIMM standard than the individual
SEPs of five firms. Two of the expired patents were owned by Micron, and
one was owned by Round Rock Research. We provide a factual summary of
these three expired patents in Table A.2 in Appendix III.

     We also calculate the value distribution of all granted LRDIMM SEPs
derived from forward citations excluding self-citations. To identify self-cita-
tions for expired SEPs, we determine whether the assignee of the citing
patent matches either the expired patent's original assignee or its assignee at
the time of expiration. If there is a match for either criterion, we exclude that
patent citation from our analysis. Figure 14 shows the distribution of value of
all granted LRDIMM SEPs, excluding self-citations.

Figure 14. Distribution of Value by Company of All
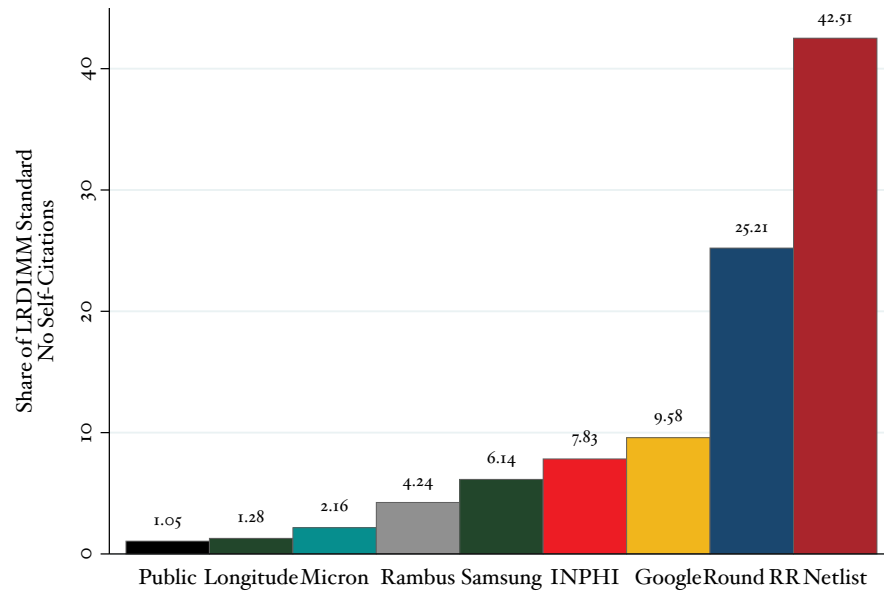Granted LRDIMM SEPs Derived from Forward
Citations, Excluding Self-Citations

Source: *PatentsView*, USPTO, http://www.patentsview.org/querydev/.
Note: Using data from PatentsView, we weighted by forward citations the distribution
of DDR4 LRDIMM SEPs across SEP holders after removing self-citations. "Public"
denotes the totality of expired LRIDMM SEPs.

We find that excluding self-citations substantially lowers Public's relative
share of the value of the LRDIMM standard from 4.98 percent to 1.05
percent.

     In sum, we can estimate the portion of the value of the technology in
JEDEC's LRDIMM standard that is in the public domain by including
expired SEPs in our forward-citation analysis.

VI. Using the Bargaining-Range Framework
to Analyze a Reasonable Royalty for
### Netlist's LRDIMM SEP Portfolio

In this part, we analyze reasonable royalties for Netlist's LRDIMM SEP portfolio within the bargaining-range framework.[123] We first summarize the economic principles underlying the bargaining-range framework. We then apply those principles to define the lower and upper boundaries of the bargaining range for Netlist's LRDIMM SEP portfolio. Using our calculated value of the surplus created by JEDEC's LRDIMM standard and the results from our apportionment analysis in Part V, we then apply the bargaining-range framework to calculate, for a hypothetical RAND offer of $12 per unit, how Netlist and a potential implementer of the LRDIMM standard would divide the surplus resulting from a voluntary transaction to license Netlist's portfolio of patents essential to the LRDIMM standard.

A.   *Defining the Range of Reasonable Royalties for Netlist's LRDIMM SEP Portfolio*

The calculation of a reasonable royalty begins with identifying each party's negotiating position in a hypothetical negotiation occurring on the eve of first infringement. In a hypothetical negotiation, the licensor and licensee negotiate royalties within the bargaining range, which is defined by the licensor's minimum willingness to accept and the licensee's maximum willingness to pay. If the licensee's maximum willingness to pay exceeds the licensor's minimum willingness to accept, the negotiation will be successful and the two parties will negotiate on a royalty that falls within that bargaining range. However, if the licensee's maximum willingness to pay is less than the licensor's minimum willingness to accept, the bargaining range will be negative, and there will be no surplus over which to bargain. Put differently, if the bargaining range is negative, there will be no voluntary exchange possible, and the parties will fail to negotiate a license. For the purposes of our analysis, we assume that the bargaining range for Netlist's LRDIMM SEPs is positive. Figure 15 below shows a graphic representation of the bargaining range.

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[123]   The original explication of this framework is J. Gregory Sidak, *Bargaining Power and Patent Damages*, 19 Stan. Tech. L. Rev. 1 (2015), which the Federal Court of Canada adopted in its March 2, 2017 decision in Airbus Helicopters, S.A.S. v. Bell Helicopter Textron Canada Limitée, 2017 F.C. 170 (Can.), http://decisions.fct-cf.gc.ca/fc-cf/decisions/en/223697/1/document.do. Precursors to that article include Sidak, *The Meaning of FRAND, Part I: Royalties*, *supra* note 91, at 931; Jerry A. Hausman, Gregory K. Leonard & J. Gregory Sidak, *Patent Damages and Real Options: How Judicial Characterization of Noninfringing Alternatives Reduces Incentives to Innovate*, 22 Berkeley Tech. L.J. 825, 832 (2007).

Figure 15. The Bargaining Range for a Reasonable
Royalty in a Hypothetical Negotiation
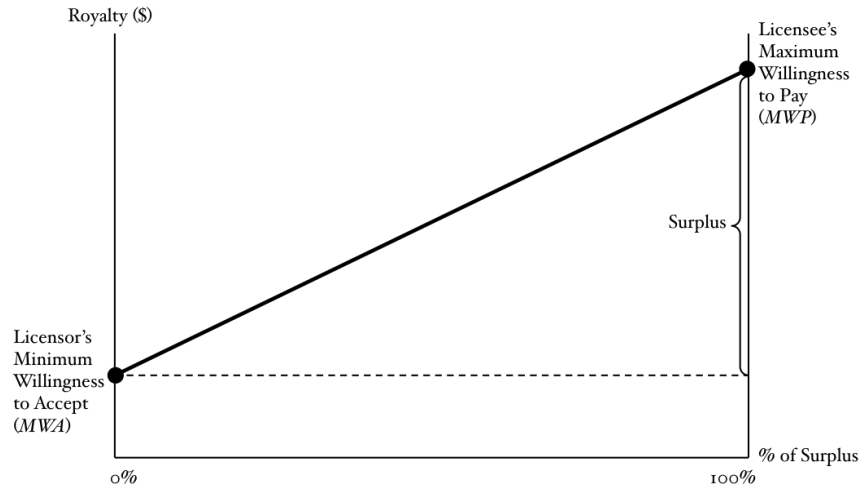
To determine the lower bound and the upper bound on the bargaining range
of a reasonable royalty, one must further examine as many of the *Georgia-
Pacific* factors as are relevant and for which facts or data exist.[124] The analysis
of the bargaining range for a reasonable royalty for Netlist's LRDMIM SEP
portfolio can be reduced to two economic questions: (1) "What is Netlist's
minimum willingness to accept?" and (2) "What is the licensee's maximum
willingness to pay?"

1.   *Identifying Netlist's Minimum Willingness to Accept*

One factor affecting a licensor's minimum willingness to accept is its "outside
option"—that is, its best alternative to striking a voluntary license agreement
with the prospective licensee. If the licensor does not practice its patents,
that outside option consists of the next-most remunerative means of mone-
tizing the patents in suit. For example, one might reasonably assume, as
Judge James Robart did in his decision in *Microsoft Corp. v. Motorola, Inc.*, that
the licensor's outside option would be to license the patents in suit through
a patent pool.[125] If, however, there is not sufficient evidence to defend the
opinion that the licensor could derive licensing revenue from a patent pool,
it might be necessary to assume that the licensor's minimum willingness to

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[124]   For a detailed discussion of how each *Georgia-Pacific* factor might influence the ultimate point royalty
within the bargaining range, see Sidak, *Bargaining Power and Patent Damages*, *supra* note 123, at 17–20.

[125]   No. C10-1823JLR, 2013 WL 2111217, at *74–75 (W.D. Wash. Apr. 25, 2013) (Robart, J.); *see also* J. Gregory
Sidak, *Tournaments and FRAND Royalties*, 1 CRITERION J. ON INNOVATION 101, 104 (2016).

accept is zero. For the purposes of our analysis, we assume conservatively that Netlist is unable to derive licensing revenue from any outside option, such that Netlist's minimum willingness to accept is assumed to be zero.

Another factor affecting a licensor's minimum willingness to accept is whether the licensor has incurred a participation cost—that is, the sunk cost that a firm incurs to participate in standard setting. In expected-value terms, a participant in standard setting must earn a competitive risk-adjusted return on its investment. If a prospective participant cannot recoup even its participation costs, then it will not participate in collective standard setting. Moreover, if the division of the gains from trade in the first round of an economic game is so unfavorable to the SEP holder that, contrary to his initial expectation, he cannot recoup even his sunk costs and a competitive risk-adjusted return, then, barring some intervening event that would provide him an ancillary revenue stream, that SEP holder will forgo the *next* round of standard setting altogether in favor of a different mechanism for monetizing his patents. Thus, a licensor's costs of participating in the standard-setting process will necessarily set a lower bound on its minimum willingness to accept in a voluntary license negotiation with an implementer that seeks to use the licensor's SEPs.

For ease of exposition, however, we assume conservatively that Netlist has not incurred any costs to participate in JEDEC's standard setting. In making that assumption, we of course do not assert that it is true, either for Netlist or for SEP holders generally. In other words, we define the lower bound on the bargaining range for Netlist's LRDIMM SEP portfolio to be $0.

2. *Identifying a Licensee's Maximum Willingness to Pay*

To identify a licensee's maximum willingness to pay, one must isolate the value created by the precise footprint of the patented invention. The incremental value that each patent in suit creates for the licensee must be calculated relative to the next-best noninfringing substitute available to the licensee. Without expert technical testimony identifying the next-best substitute technology, or without fact testimony from an appropriately qualified employee of the licensor or licensee, it might be difficult for an expert witness on damages to defend an opinion on a defendant's maximum willingness to pay. However, if there is available for the patented technology direct evidence of the licensee's *demonstrated* willingness to pay—such as royalties voluntarily set in comparable licenses for the patents in suit or market prices actually paid for products practicing those patents—then an expert witness

can use those data to form a conservative estimate of the upper bound on the licensee's maximum willingness to pay.[126]

In Part V, we used observed prices from historical sales of representative RDIMM and LRDIMM products to calculate the net incremental value that purchasers of memory modules derive from JEDEC's LRDIMM standard. Put differently, we calculated those consumers' demonstrated willingness to pay for LRDIMM-compliant products above and beyond their willingness to pay for RDIMM-compliant products. Because our hedonic model uses actual price data, we cannot observe directly the licensee's theoretical *maximum* willingness to pay. Instead, we observe its *demonstrated* willingness to pay, which is less than or equal to its maximum willingness to pay. A licensee's demonstrated willingness to pay of course will not exceed that of downstream consumers, as it would be nonsensical for a willing licensee to earn negative profits by voluntarily selling LRDIMM products at a price that is lower than what it has paid to obtain a license to the necessary SEPs. Thus, the net incremental value of JEDEC's LRDIMM standard that is attributable to Netlist's LRDIMM SEP portfolio is a conservative estimate of the upper bound on the licensee's maximum willingness to pay for Netlist's LRDIMM SEP portfolio.

Put differently, we define the upper bound of the bargaining range for Netlist's LRDIMM SEP portfolio as $34.27.[127] Thus, we define the bargaining range for a reasonable royalty for Netlist's LRDIMM SEP portfolio to extend from $0 to $34.27.

3.   *The Negative Bargaining Range*

For the purposes of our analysis, we have assumed that the bargaining range for Netlist's LRDIMM SEP portfolio is positive. In other words, we assume that the licensee's maximum willingness to pay exceeds Netlist's minimum willingness to accept. However, it is possible that the licensee's maximum willingness to pay is less than Netlist's minimum willingness to accept, such that the bargaining range is negative. Figure 16 below shows a graphic representation of the negative bargaining range.

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[126] *See* Sidak, *Bargaining Power and Patent Damages*, *supra* note 123, at 13 ("In practice, the facts and data of the case might enable the finder of fact to approximate the upper bound of the bargaining range by the licensee's actually observed willingness to pay, which I call its *demonstrated* willingness to pay. Because the licensee's demonstrated willingness to pay is, by definition, less than or equal to the licensee's true maximum willingness to pay, the bargaining range is a conservative estimate of the surplus generated by a successful bargain." (emphasis in original)).

[127] We use the incremental value of the LRDIMM standard that is attributable to Netlist's LRDIMM SEP portfolio, which we calculated after excluding self-citations in Part V.

Figure 16. The Negative Bargaining Range
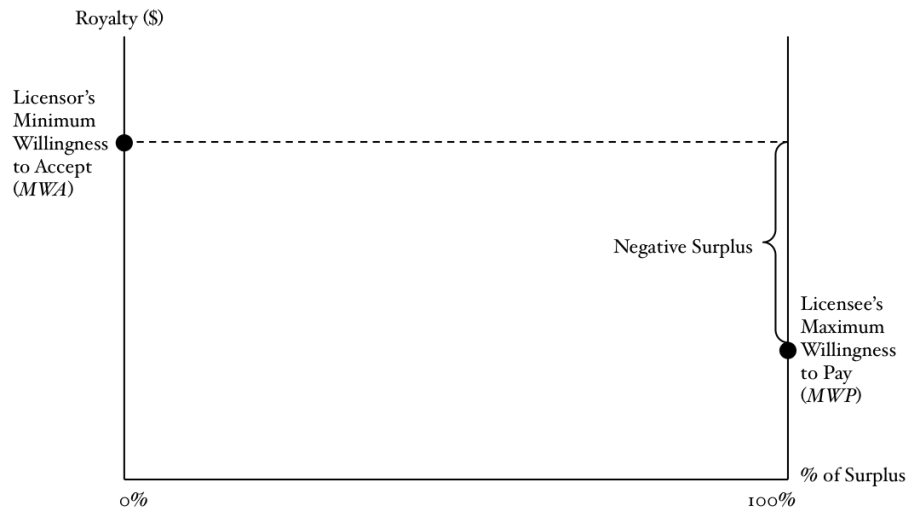
As Figure 16 shows, when the licensee's maximum willingness to pay is below the licensor's minimum willingness to accept, there is a negative surplus. In effect, there is no positive surplus over which to bargain; there are no "gains from trade." Nonetheless, a patent holder in the United States is entitled by statute to no less than a reasonable royalty if its patent is infringed,[128] which in economic terms means that the patent holder is entitled to no less than its minimum willingness to accept. Thus, when there is a negative bargaining range, full compensation of the patent holder for patent infringement requires the infringer to pay the patent holder an amount not less than the patent holder's minimum willingness to accept, even if it exceeds the licensee's maximum willingness to pay.

B.   Calculating the Division of Surplus Between Netlist and a Potential Implementer of the LRDIMM Standard on the Basis of a Hypothetical $12 Per-Unit Royalty

In economic terms, the difference between the licensor's minimum willingness to accept and the licensee's maximum willingness to pay defines the economic surplus, or gains from trade, from a successful negotiation for a voluntary license for the patents in suit.[129] Put differently, the bargaining range measures the value created by a successful licensing transaction. Thus

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[128]   35 U.S.C. § 284 ("Upon finding for the claimant the court shall award the claimant damages adequate to compensate for the infringement, *but in no event less than a reasonable royalty* for the use made of the invention by the infringer, together with interest and costs as fixed by the court." (emphasis added)).

[129]   *See* Jack Hirshleifer, Amihai Glazer & David Hirshleifer, Price Theory and Applications: Decisions, Markets, and Information 203–04 (Cambridge Univ. Press 7th ed. 2005).

arises the next economic question: how would the licensor and licensee split the gains from trade?

The ultimate royalty that results between the two parties will depend on the relative bargaining power of the parties, which in turn will depend on factors such as each party's discount rate (that is, each party's cost of capital) and its need for liquidity, among others.[130] In the interest of generality and simplicity of exposition, we refrain in our analysis from making assumptions regarding Netlist's relative bargaining power vis-à-vis any specific implementer and instead use a fixed hypothetical royalty to demonstrate the division of surplus between the two parties.

Suppose (strictly for purposes of exposition) that Netlist offered an implementer seeking to practice JEDEC's LRDIMM standard a license with a royalty of $12 per unit. Figure 17 below shows graphically the bargaining range for a RAND royalty that a hypothetical implementer owes for its use of Netlist's LRDIMM SEP portfolio. Figure 17 also shows the division of surplus that would occur between Netlist and that implementer under Netlist's hypothetical offer of a $12 per-unit rate.

Figure 17. The Bargaining Range for a RAND Royalty for
a Licensee's Use of Netlist's LRDIMM SEP Portfolio, with
Netlist's Minimum Willingness to Accept Set Equal to Zero
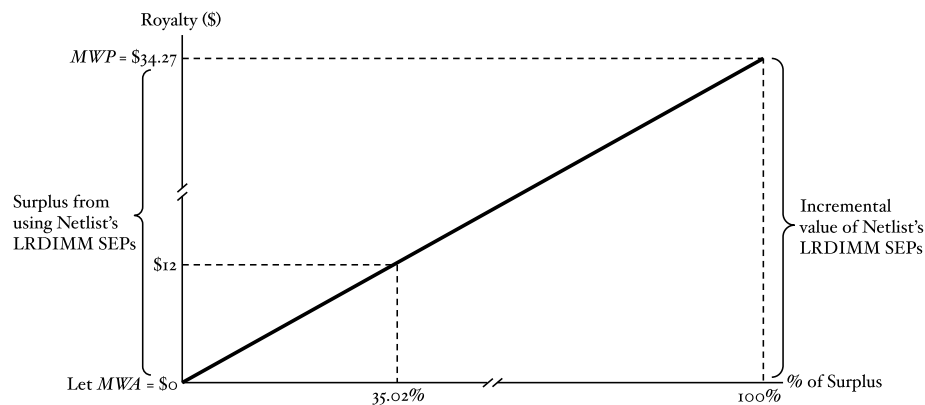
Source: Authors' original analysis.

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[130] *See* Sidak, *Bargaining Power and Patent Damages*, *supra* note 123, at 22 ("Relative bargaining power depends not only on the overall size of the benefit that each party expects, but also on the benefit from agreeing to a contract at a particular time (versus the possible benefit from agreeing to a contract at a later time). Therefore, the party that suffers least from delaying an agreement—that is, the party that is most patient—will have more bargaining power. . . . [T]he party with the lower discount rate will have more bargaining power because it suffers less from a delay in reaching an agreement."); *see also* Robert Gibbons, Game Theory for Applied Economists 68–71 (Princeton Univ. Press 1992).

The diagonal line in Figure 17 represents all of the possible royalty outcomes along the bargaining range, which can be summarized in the following equation:

$$Royalty = MWA + [s \times (MWP - MWA)], \quad\quad (7)$$

where *MWA* is Netlist's minimum willingness to accept to license its LRDIMM SEP portfolio, *MWP* is the licensee's maximum willingness to pay for Netlist's contribution to JEDEC's LRDIMM standard, and *s* is the percentage of the surplus captured by the licensor, Netlist. The vertical distance between *MWP* and *MWA* represents the total surplus created by a voluntarily negotiated licensing agreement between Netlist and its licensee; the two parties will then divide that surplus.

As we explained earlier, we conservatively set *MWA* equal to $0. Thus, we can simplify Equation 7 to

$$Royalty = s \times MWP. \quad\quad (8)$$

Given a hypothetical $12 per-unit royalty and the licensee's maximum willingness to pay of $34.27, *s* equals $12 ÷ $34.27 = 0.3502. That is, under the conservative assumptions made above, Netlist's $12 per-unit rate would capture no more than 35.02 percent of the surplus per unit that the proposed RAND licensing transaction would generate. Conversely, that license offer at $12 per unit would enable the implementer to capture $22.27 (or 64.98 percent) of the surplus per unit from that transaction.

Suppose instead that Netlist's minimum willingness to accept to license its LRDIMM SEP portfolio to an implementer exceeded zero (because Netlist had a viable outside option to entering into the license). In that case, Netlist's *MWA* would be a positive *y*-intercept in the bargaining-range graph, as Figure 18 below shows.

Figure 18. The Bargaining Range for a RAND Royalty for
a Licensee's Use of Netlist's LRDIMM SEP Portfolio, with
Netlist's Minimum Willingness to Accept Exceeding Zero
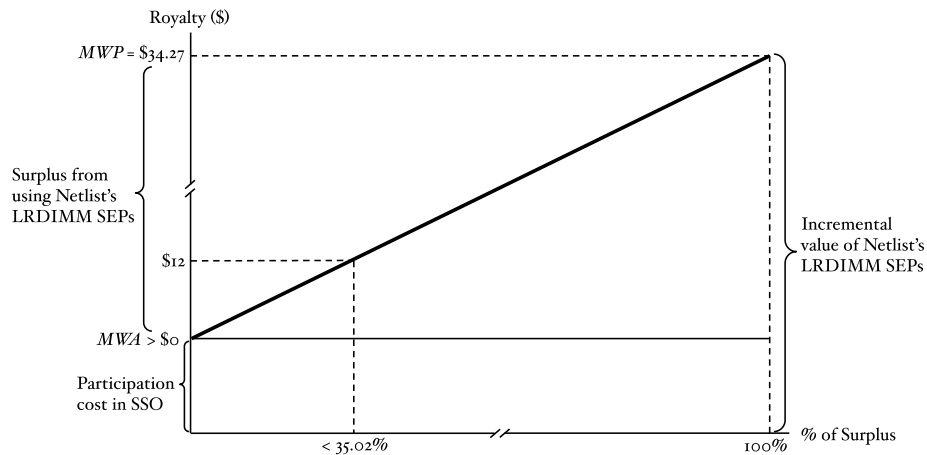
Source: Authors' original analysis.

Because *MWA* exceeds zero, the total surplus from using Netlist's LRDIMM SEPs—that is, the distance between *MWA* and *MWP*—in Figure 18 has decreased relative to the total surplus in Figure 17. That is, for a given royalty, the licensor captures the greatest portion of the surplus when its minimum willingness to accept is zero. As its minimum willingness to accept increases (while keeping the per-unit royalty constant), the surplus from the voluntary license transaction decreases and the proportion of the surplus that the licensor captures also decreases. As Figure 18 shows, if Netlist has a positive *MWA* (for example, because it has a viable outside option to the licensing transaction or because it has a positive cost of participating in JEDEC's development and setting of the LRDIMM standard), Netlist will necessarily capture *less than* 35.02 percent of the value that its SEPs contributed to JEDEC's LRDIMM standard.

   When *MWA* exceeds zero, one no longer can use Equation 8 to calculate the value of *s*. Instead, one must rearrange Equation 7 and supply the appropriate values. (One can rearrange Equation 7 as: *s* = (*Royalty* – *MWA*) ÷ (*MWP* – *MWA*)). Table 4 below reports the respective shares of surplus that would flow to Netlist and to a potential licensee under different assumptions regarding the level of Netlist's minimum willingness to accept.

Table 4. The Division of the Surplus Created by a Voluntarily Negotiated
Licensing Agreement Between Netlist and a Potential Licensee

	RAND Royalty	Netlist's Hypothetical MWA	Licensee's MWP	Netlist's Portion of the Surplus	Licensee's Portion of the Surplus
				$([A] - [B])$ $\div ([C] - [B])$	$1 - [D]$
	[A]	[B]	[C]	$= [D]$	$= [E]$
[1]	$12.00	$0.00	$34.27	35.02%	64.98%
[2]	$12.00	$3.00	$34.27	28.78%	71.22%
[3]	$12.00	$6.00	$34.27	21.22%	78.78%
[4]	$12.00	$9.00	$34.27	11.87%	88.13%
[5]	$12.00	$12.00	$34.27	0.00%	100.00%

Source: Authors' original analysis.

In Table 4, rows [1] through [5] report the share of the surplus that would flow to Netlist if it receives a hypothetical RAND royalty of $12 per unit, assuming that Netlist has a different minimum willingness to accept in each row. If Netlist's *MWA* were zero (as row [1] reports), 35.02 percent of the surplus would flow to Netlist and 64.98 percent of the surplus would flow to the licensee. As Netlist's *MWA* approaches $12, its share of the surplus approaches zero. Figure 19 below shows a graphic representation of that relationship between Netlist's *MWA* and its share of the surplus from a voluntarily negotiated license agreement on RAND terms.

Figure 19. The Division of Surplus Between Netlist and a Potential Licensee,
with Different Values for Netlist's Minimum Willingness to Accept
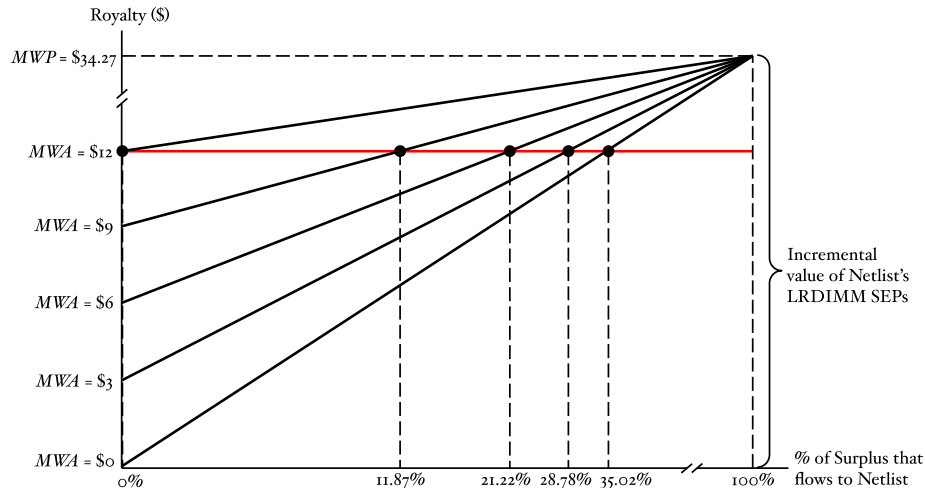
Source: Authors' original analysis.

In Figure 19, each diagonal line represents the bargaining range for each alternative value of Netlist's minimum willingness to accept. The red line represents a constant per-unit RAND royalty of $12. Figure 19 shows that, as Netlist's *MWA* increases from $0 to $12, Netlist's proportion of the surplus (which the intersection between each diagonal line and the red line defines on the *x*-axis) decreases from 35.02 percent to 0 percent.

VII. Admissibility, Apportionment, and the Unreliable Alternative of *Ex Ante* Incremental Valuation

Our hedonic price analysis methodology is admissible as expert testimony under the Federal Rules of Evidence. It also complies with the Federal Circuit's apportionment requirement for patent damages. In contrast, an analysis of *ex ante* incremental value of the patents in suit at the time of standard adoption is methodologically flawed on economic grounds and consequently irrelevant, unhelpful, and unreliable in an evidentiary sense.

A. Why the Hedonic Price Methodology Is Admissible

Our hedonic price methodology is admissible as expert economic testimony. As of the date of publication of this article in 2017, the proper lens through which to analyze the admissibility of expert testimony employing hedonic

price estimation is not the formulaic checklist in *Daubert*.[131] Rather, it is the Supreme Court's specific observation, also in *Daubert*, that "[s]ome propositions . . . [are] too new . . . to be published."[132] Justice Stephen Breyer amplified this theme for the Supreme Court in *Kumho*, noting that the novelty of a methodology does not render the methodology inadmissible as expert testimony.[133] *Daubert*, Justice Breyer wrote, "made clear that its list of factors was meant to be helpful, not definitive."[134] "It might not be surprising in a particular case," he elaborated, "that a claim made by a scientific witness has never been the subject of peer review, for the particular application at issue may never previously have interested any scientist."[135] Every useful idea was once novel and therefore necessarily unsubstantiated in the preexisting scholarly literature. Justice Breyer wrote in *Kumho* that what matters for the admissibility of expert testimony is "intellectual rigor."[136]

As we explained in the introduction to this article, we are not aware of any prior patent-infringement case in which a federal district court (or, for that matter, the ITC or an arbitral panel) has admitted into evidence expert economic testimony that relies on econometric estimation of hedonic prices to set a reasonable royalty. That absence of precedent is the factual basis for our claim to having made a novel contribution to the calculation of a RAND or FRAND royalty for a multicomponent product.

However, in patent-infringement cases, courts definitely have admitted into evidence expert economic testimony that relies on conjoint analysis, which uses statistical techniques similar to those used in hedonic price analysis. Although in some cases courts have found the results of conjoint analysis to be inadmissible, those rulings resulted from the expert's improper data collection and survey techniques, not from any problem concerning the integrity of the econometric model itself.[137] Indeed, the Supreme Court has accepted that regression analysis (when properly implemented) is a cornerstone of empirical research and an admissible scientific methodology for performing empirical estimations.[138]

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[131] Daubert v. Merrell Dow Pharm., Inc., 509 U.S. 579, 592–95 (1993).

[132] *Id.* at 593.

[133] Kumho Tire Co. v. Carmichael, 526 U.S. 137, 151 (1999).

[134] *Id.*

[135] *Id.*; *see also* Stephen Breyer, *Introduction*, *in* Reference Manual on Scientific Evidence 1, 6 (Fed. Judicial Ctr. 3d ed. 2011).

[136] *Kumho*, 526 U.S. at 152.

[137] *See* J. Gregory Sidak & Jeremy O. Skog, *Using Conjoint Analysis to Apportion Patent Damages*, 25 Fed. Cir. Bar J. 581, 586–91, 599–609 (2016) (discussing district court orders concerning the admissibility of expert testimony on conjoint analysis surveys in patent-infringement litigation).

[138] *See, e.g.*, Halliburton Co. v. Erica P. John Fund, Inc., 134 S. Ct. 2398, 2415 (Roberts, C.J.) (explaining how an event study, which employs regression analysis, is useful as evidence for purposes of certifying a class); Thornburg v. Gingles, 478 U.S. 30, 53 n.20 (1986) (calling regression analysis "standard in the literature for the analysis of racially polarized voting").

Furthermore, since at least 2010 the federal district courts have admitted into evidence expert economic testimony that relies on econometric estimations of hedonic prices to support certification of a proposed class and to calculate damages suffered by members of a class. In 2017, the Ninth Circuit ruled in a food-labeling class action case that the district court did not abuse its discretion by allowing the plaintiffs, at the class-certification stage, to "propose to measure the classwide price premium attributable to their theory of liability using . . . hedonic regression analysis to calculate the price premium attributable to the '100% Natural' label" on bottles of Wesson cooking oils that allegedly contained bioengineered ingredients that were not natural.[139] The Ninth Circuit called hedonic regression analysis a "well established damages model[]."[140]

As we have shown in this article, it is more than possible to *propose* a methodology to measure consumer value of product features through the use of a hedonic price regression. If the data are available, it is actually possible to perform that measurement with scientific rigor.

B. *Why the Hedonic Price Methodology Complies with the Federal Circuit's Apportionment Requirement*

In *Ericsson v. D-Link*, decided in 2014, the Federal Circuit emphasized that, "[a]s with all patents, the royalty rate for SEPs must be apportioned to the value of the patented invention."[141] The court said:

> When dealing with SEPs, there are two special apportionment issues that arise. First, the patented feature must be apportioned from all of the unpatented features reflected in the standard. Second, the patentee's royalty must be premised on the value of the patented feature, not any value added by the standard's adoption of the patented technology.[142]

The Federal Circuit added that these two steps are necessary to ensure that the damages award will reflect "the incremental value that the patented *invention* adds to the product, not any value added by the standardization."[143] The following year, in *Commonwealth Scientific and Industrial Research Organisation (CSIRO) v. Cisco Systems, Inc.*, the Federal Circuit emphasized that the same principles apply to all SEPs, regardless of whether they are subject to a

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[139] *See* Briseno v. ConAgra Foods, Inc., No. 15-55727, 2017 WL 53421, at *2 (9th Cir. Jan. 3, 2017).

[140] *Id.*

[141] Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1232 (Fed. Cir. 2014).

[142] *Id.*

[143] *Id.* (emphasis in original).

FRAND commitment.[144] Our methodology provides a rigorous means for implementing the Federal Circuit's apportionment requirement.[145]

First, by comparing the value of the standard in question with the next-best alternative standard, our hedonic regression analysis separates the value of the technologies included in the standard from the value created by a having a standard of any sort in the first place. We define the value of standardization as the value of the agreement to implement a unified standard. That value arises from (1) the reduction in transaction costs for implementers of the standard and for SEP holders, and (2) the network effects generated by the substitutability of and interoperability between standard-compliant products. Customers prefer LRDIMMs to RDIMMs because of the more technologically advanced features (such as the reduced load on the computer server's central processing unit (CPU) at higher densities) that the LRDIMM standard supports, rather than the need to agree on a common standard and the benefit from stimulating network effects. Therefore, the difference in value between the two standards (LRDIMM and RDIMM) reflects the value of the new technologies included in the LRDIMM standard, rather than the value of standardization—which the RDIMM standard (if not a predecessor technology standardized by JEDEC, such as DIMM) had already achieved for memory modules.

Second, our methodology separates the value of the SEPs in suit from the incremental value of JEDEC's LRDIMM standard. We do so by identifying the universe of active U.S. patents that companies have declared to be essential to the LRDIMM standard. We then apportion the relative value that a given SEP holder's LRDIMM portfolio contributes to the total value of JEDEC's LRDIMM standard by calculating a weighted patent-citation score for each SEP. That is, we weight the value of each LRDIMM SEP by its forward citations to account for that SEP's value relative to other SEPs for the LRDIMM standard (including expired SEPs that are in the public domain). By relying on that apportionment methodology, we assess the value of an individual SEP holder's contribution to the incremental value that JEDEC's LRDIMM standard has created.

In sum, our methodology assesses the value of the portfolio of the asserted LRDIMM SEPs "resulting not from the value added by the standard's widespread adoption, but only from the technology's superiority."[146] In principle, we could apply a similar methodology to measure a reasonable royalty for any patented technology included in a multicomponent product, even if that technology were not essential to any industry standard.

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[144] 809 F.3d 1295, 1304–05 (Fed. Cir. 2015).
[145] The intuition of our approach appears in nontechnical terms in Sidak, *The Value of a Standard Versus the Value of Standardization*, *supra* note 3.
[146] *CSIRO*, 809 F.3d at 1304.

C. Why the Ex Ante *Incremental Value Methodology Is Irrelevant, Unhelpful, and Unreliable*

Our methodology does not purport to implement the *ex ante* incremental value methodology that some academics, practitioners, and government officials have argued should guide the determination of RAND (and FRAND) royalties. For example, the Federal Trade Commission said in 2011 that "[a] definition of RAND based on the *ex ante* value of the patented technology at the time the standard is chosen is necessary for consumers to benefit from competition among technologies to be incorporated into the standard."[147] Consequently, the FTC reasoned, "[c]ourts should cap the [RAND] royalty at the incremental value of the patented technology over alternatives available at the time the standard was chosen."[148] That recommendation, although embraced in some academic articles,[149] violates the Federal Circuit's subsequent guidance in *Ericsson v. D-Link* for computing a RAND or FRAND royalty.

Ericsson v. D-Link requires that damages for the infringement of SEPs reflect the "incremental value that the patented *invention* adds to the product, not any value added by the standardization of that technology."[150] However, the Federal Circuit has never limited the RAND royalty to the incremental value of the SEP over the next-best, alternative technology that the SSO could have chosen to implement the standard. Similarly, in determining a reasonable royalty for patent infringement, courts use the term *ex ante* when referring to a negotiation on the eve of first infringement.[151] The Federal

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[147] Federal Trade Commission, The Evolving IP Marketplace: Aligning Patent Notice and Remedies with Competition 22–23 (2011) (emphasis added).

[148] *Id.* at 23.

[149] *See* Dennis W. Carlton & Allan L. Shampine, *An Economic Interpretation of FRAND*, 9 J. Competition L. & Econ. 531, 545 (2013) ("A 'reasonable' royalty . . . in the context of FRAND and an SSO is a royalty . . . that would have been negotiated *ex ante*, before the patented technology at issue had been adopted into the standard and prior to the licensee incurring sunk costs. The maximum royalty *ex ante* is based on the incremental value that the technology brings to the licensee compared with the next-best alternative available. No firm would pay more than that royalty in an *ex ante* negotiation when an alternative is available."); Gregory K. Leonard & Mario A. Lopez, *Determining RAND Royalty Rates for Standard-Essential Patents*, 29 Antitrust, Fall 2014, at 86, 87 ("The definition of RAND can be . . . refined to be the *ex ante* incremental value of the SEP, which is the additional value provided by the SEP over the next-best substitute technology."); Janusz Ordover & Allan Shampine, *Interpreting the FRAND Commitment*, Antitrust Source, Oct. 2014, at 1, 8 ("[T]he *ex ante* framework asks what is the incremental value of the patented technology relative to the alternatives available prior to the standard being set. The goal is to preserve the benefits of any competition that was actually or potentially present prior to the standard being set.").

[150] Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1232 (Fed. Cir. 2014) (emphasis in original).

[151] For example, in *Lucent Technologies, Inc. v. Gateway, Inc.*, the Federal Circuit said:

> Litigants routinely adopt several approaches for calculating a reasonable royalty. . . . The . . . more common approach, called the hypothetical negotiation[,] . . . attempts to ascertain the royalty upon which the parties would have agreed had they successfully negotiated an agreement just before infringement began. . . . The hypothetical

Circuit has *not* adopted a different approach in cases involving the infringement of SEPs.

In other words, contrary to the common admonition in the academic literature advocating the patent-holdup conjecture, the Federal Circuit has not moved the date of the hypothetical negotiation to the date of the standard's adoption. In *Ericsson v. D-Link*, the Federal Circuit explicitly said that "[o]ne amicus suggests that the jury always should be told to place the date of the hypothetical negotiation as of the date of the adoption of the standard (if that date predates the infringement) so as to discount any value added by the standardization."[152] However, because "D-Link did not request any such instruction," the Federal Circuit "[did] not address whether shifting the timing of the hypothetical negotiation is either appropriate or necessary."[153] To this day the Federal Circuit still has not embraced the "*ex ante* incremental value methodology" that some professors advocate.

Lower courts have similarly declined to adopt the *ex ante* incremental value approach in determining damages for the infringement of SEPs. *Innovatio IP Ventures*, decided in 2013 by Judge James Holderman of the U.S. District Court for the Northern District of Illinois, is the only reported case in the United States in which a court explicitly set the hypothetical negotiation at a time before the SSO had adopted the technology covered by the patent in suit into the standard.[154] However, in *Innovatio* the date of the standard's adoption happened to coincide with the date of first infringement.[155] The parties did not dispute the date of the hypothetical negotiation.

Moreover, Judge Holderman never attempted in *Innovatio* to identify the incremental value of the SEPs in suit over the next-best alternative available at the time of the standard's adoption. And for good reason. As Judge James Robart of the U.S. District Court for the Western District of Washington emphasized later in 2013 in *Microsoft Corp. v. Motorola Inc.*, the *ex*

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negotiation tries, as best as possible, to recreate the *ex ante* licensing negotiation scenario and to describe the resulting agreement.

580 F.3d 1301, 1324 (Fed. Cir. 2009); *see also* Aqua Shield v. Inter Pool Cover Team, 774 F.3d 766, 770 (Fed. Cir. 2014) (Taranto, J.) (citing *Lucent v. Gateway*, 580 F.3d at 1324–25); Riles v. Shell Expl. & Prod. Co., 298 F.3d 1302, 1313 (Fed. Cir. 2002) ("A reasonable royalty determination for purposes of making a damages evaluation must relate to the time infringement occurred, and not be an after-the-fact assessment."); On Track Innovations Ltd. v. T-Mobile USA, Inc., 106 F. Supp. 3d 329, 408–10 (S.D.N.Y. 2015) (discussing the relevance of post-infringement information to the determination of a reasonable royalty for patent infringement); Wonderland Nurserygoods Co., Ltd. v. Thorley Indus. LLC, No. 2:13-cv-00387, 2015 WL 6669154, at *2 (W.D. Penn. Oct. 30, 2015) ("Again, '[a] "reasonably royalty" contemplates a hypothetical negotiation between the patentee and the infringer at a time *before the infringement began*.' Rather than focusing on this ex-ante value of the patent, however, Mr. Chase analyzes the expost [sic] value of the patent by considering the inventory that Thorley acquired *after* infringement began." (citing *Riley*, 298 F.3d at 1311) (internal citations omitted) (emphasis in original)).

[152] *Ericsson v. D-Link*, 773 F.3d at 1234 n.10.

[153] *Id.*

[154] *In re* Innovatio IP Ventures, LLC Patent Litig., No. 11 C 9308, 2013 WL 5593609, at *8 (N.D. Ill. Oct. 3, 2013).

[155] *Id.*

ante incremental value approach "lack[s] . . . real-world applicability."[156] The methodology would require the court to measure the value of every SEP, to identify the alternatives available at the time of the SSO's discussion of the standard, and then to determine their respective values. That task would be absurdly fact-intensive, time-consuming, costly, and speculative. Ultimately, it would be impossible to perform in practice.

In almost all cases, expert testimony relying on the *ex ante* incremental value approach will fail to satisfy the requirements for admissible evidence. Under the Federal Rules of Evidence, evidence must be relevant to be admissible. Rule 401 of the Federal Rules of Evidence specifies two necessary conditions that must hold for evidence to be relevant.[157] First, consistent with Rule 401(a), the evidence must be probative, meaning that the evidence must make the factual proposition more (or less) likely than it would be without the evidence.[158] Second, consistent with Rule 401(b), the evidence must be material, meaning that there must be a link between the factual proposition that the evidence tends to establish and the legal test that the evidence seeks to satisfy.[159] According to these criteria, the *ex ante* incremental value approach is irrelevant: evidence derived from the approach is neither probative nor material.

Indeed, expert testimony relying on the *ex ante* incremental value approach will typically be substantially more prejudicial than probative. Rule 703 of the Federal Rules of Evidence specifies that, even if the facts or data that form the basis of an expert's opinion "would otherwise be inadmissible, the proponent of the opinion may disclose them only if their probative value in helping the jury evaluate the opinion substantially outweighs their prejudicial effect."[160] The *ex ante* incremental value approach lacks probative value because it is unworkable in practice.

In addition, expert testimony regarding the *ex ante* incremental value approach is unhelpful. Consistent with Rule 702 of the Federal Rules of Evidence, expert testimony must be helpful to be admissible.[161] In cases regarding RAND royalties for SEPs, for example, commentators have debated the evidentiary value of the patent-holdup and royalty-stacking conjectures. As a result, the Federal Circuit has said that those conjectures must be rejected unless they are tied to specific facts of the case.[162] The same

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[156] No. C10-1823JLR, 2013 WL 2111217, at *13 (W.D. Wash. Apr. 25, 2013) (Robart, J.).

[157] FED. R. EVID. 401.

[158] *Id.* 401(a).

[159] *Id.* 401(b).

[160] *Id.* 702; *see also* Micro Chem., Inc. v. Lextron, Inc., 317 F.3d 1387, 1391 (Fed. Cir. 2003) ("In 2000, Rule 702 was amended in response to *Daubert* and cases applying it.").

[161] FED. R. EVID. 702.

[162] *See, e.g.*, Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1234 (Fed. Cir. 2014); Commonwealth Sci. & Indus. Research Org. v. Cisco Sys., Inc., 809 F.3d 1295, 1302 (Fed. Cir. 2015). More generally, the Supreme Court has said: "When an expert opinion is not supported by sufficient facts to validate it in the eyes of

commentators who propound those two conjectures of market failure also advocate that a RAND royalty for an SEP should be based on the *ex ante* incremental value.[163] Unsurprisingly, that argument faces the same problem of nonfalsifiability as the patent-holdup and royalty-stacking conjectures. Because the *ex ante* incremental value approach requires data that typically do not exist, the exercise cannot be empirical. Thus, it cannot be helpful to the finder of fact.

We do not attempt in this article to apply a theoretical approach that the Federal Circuit has never endorsed and that is impossible to administer in the real world. Instead, we have explained and applied in this article a more rigorous economic methodology that is simpler to implement in practice, that relies on actual rather than nonexistent data, and that ultimately helps the finder of fact by reliably apportioning the reasonable royalty according to the individual contribution of the SEPs in suit.

A similarly irrelevant criticism is that our hedonic price methodology does not provide information about the nondiscrimination component of the RAND obligation. That complaint is true. But, to borrow the wit of the late William Baumol, so also is it true that our hedonic price analysis does not balance the budget or cure baldness. The complaint that hedonic price analysis does not inform the prohibition on "unfair discrimination" reflects a fundamental misunderstanding about the objective of hedonic price analysis in this context—namely, to estimate the incremental value of a standard above and beyond the value of the next-best option. The results of that analysis indicate a ceiling for the reasonable royalty; those empirical results do not tell us—and are not intended to tell us—whether that reasonable rate is unfairly discriminatory within the meaning of the RAND or FRAND commitment defined by contract by the specific SSO in question.[164]

Conclusion

The use of hedonic price estimation is a conceptual breakthrough in the calculation of reasonable royalties for patent infringement, both for standard-essential patents subject to a RAND or FRAND commitment and for patents that are not declared essential to any standard. Hedonic price analysis

the law, or when indisputable record facts contradict or otherwise render the opinion unreasonable, it cannot support a jury's verdict." Brooke Group Ltd. v. Brown & Williamson Tobacco Corp., 509 U.S. 209, 242 (1993).

[163] *See, e.g.*, Scott-Morton & Shapiro, *Strategic Patent Acquisitions*, *supra* note 91, at 471 ("Reasonable royalties, based on a hypothetical ex ante negotiation, should never exceed the ex ante incremental value of the patented technology. Reasonable royalties will thus be small relative to the value of the downstream product if the technology has close ex ante substitutes, especially if it covers one feature in a complex device or service.").

[164] A thorough legal and economic analysis of the nondiscrimination requirement of the RAND or FRAND commitment appears in J. Gregory Sidak, *Fair and Unfair Discrimination in Royalties for Standard-Essential Patents*, 2 Criterion J. on Innovation 301 (2017).

provides a scientifically rigorous means to satisfy the Federal Circuit's directive in *Ericsson v. D-Link* to disaggregate the value of having a standard of any sort from the incremental value of the chosen standard, and then to disaggregate further the incremental contribution that a given SEP or portfolio of SEPs makes to the overall value of that chosen standard.

The common additive form of the hedonic regression model is the most appropriate econometric model to meet that directive. When implemented in an appropriate and thoughtful way, hedonic price analysis provides an expert economic witness—and, ultimately, the finder of fact—with a reliable methodology to determine whether a given license offer satisfies the reasonableness requirement of a RAND or FRAND commitment. For similar reasons, hedonic price estimation can inform the calculation of a reasonable royalty in conventional patent litigation that does not involve standard-essential patents.

APPENDIX I. SPECIMEN OF A JEDEC
LICENSE ASSURANCE

Page 1 of 2

License Assurance/Disclosure Form

PART 1

Date: 11/22/10

Entity Name: Netlist, Inc.

Address: 51 Discovery, Suite 150

Irvine, CA 92618

IPR Contact: ████████████

Phone: ████████████

E-Mail: ████████████████

PART 2

Identify the relevant JEDEC Standard:

JC42.3   3DS   (#1655.19, #1644.96, #1655.1)

JC45   LRDIMM (#2192.62, #2192.43, #2192.58, #2192.57)

PART 3

For Issued Patents:

Patent No.: _____

Patent Name or Title: _____

Country of Issuance: United States

For Published Patent Applications:

Published Patent Application No.: 12/761,179

Patent Application Name or Title: System and Method Utilizing Distributed

Country of Filing: United States   Byte-Wise Buffers on a Memory Module

For Unpublished Patent Applications:

Subject Matter of Patent Application:

_____

_____

(See JM21P, 8.2.3 and 8.2.5 of the JEDEC Patent Policy for additional details)   27/

License Assurance/Disclosure Form

You must complete Part 4 if the entity holds a Patent or has applied for a Patent on an invention the use of which is or may be required to comply with a Standard that may result from the JEDEC Standard Activity.

PART 4

For any Essential Patent Claims held or controlled by the entity, pending or anticipated to be filed, the entity states:

___ (i) A license will be offered, without compensation, under reasonable terms and conditions that are demonstrably free of any unfair discrimination to applicants desiring to utilize the license for the purpose of implementing the JEDEC Standard; *or*

✓ (ii) A license will be offered to applicants desiring to utilize the license for the purpose of implementing the JEDEC Standard under reasonable terms and conditions that are demonstrably free of any unfair discrimination.

PART 5

For any disclosed Patent or Patent Application that contains Essential Patent Claims which, if licensed, would require a payment of royalties or other material consideration to an unaffiliated third party, provide the following information:

Name of Third Party: _____

Address: _____

_____

_____

IPR Contact: _____

Phone: _____

E-Mail: _____

AGREED, ON BEHALF OF THE ENTITY:

████████████████   _____ 11/30/2010 _____
(Signature)   (Date)

████████████████
(Name printed)

NOTE: The committee or task group has the right to request additional technical information relating to the Patent or Patent Application in order to consider workarounds and other technical alternatives.

RETURN THE COMPLETED FORM BY MAIL TO
JEDEC, ATTENTION: LEGAL DEPARTMENT,
3103 NORTH 10TH STREET, SUITE 240-S, ARLINGTON, VA 22201-2107
**OR BY E-MAIL TO JOHNK@JEDEC.ORG**

28/

Source: JEDEC.
Note: This License Assurance/Disclosure Form discloses the patent application for U.S. Patent No. 8,516,185, held by Netlist, Inc. For privacy reasons, we have redacted the name, contact information, and signature of Netlist's representative.

Appendix II. Estimating the Gini Coefficient

The Gini coefficient, "[t]he most commonly accepted measure of inequality [within a distribution],"[165] is defined as the ratio of the area between the 45-degree line and the Lorenz curve to the total area under the 45-degree line.[166] The 45-degree line represents a perfectly equal distribution of values, and the Lorenz curve represents the actual distribution of values in the data.[167] Thus, the Gini coefficient measures the extent to which the distribution in the data deviates from a hypothetical, perfectly equal distribution. In this appendix, we introduce the trapezoid-rule methodology that economists often employ to calculate the Gini coefficient. We also introduce a simpler alternative methodology that is appropriate for our data and that we have used to calculate the reported Gini coefficients in Part V.E above.

Figure A1. Graphic Representation of the Gini Coefficient
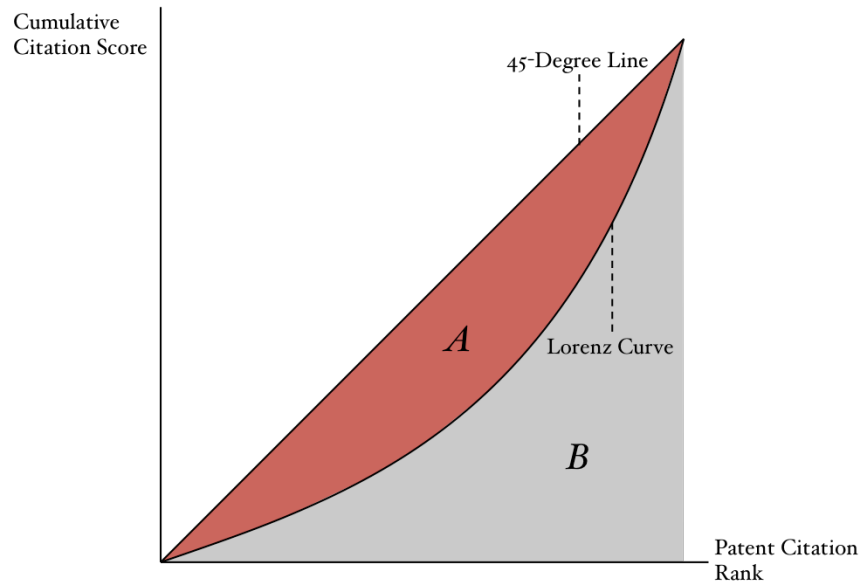
Source: Authors' original figure.

In Figure A1, the vertical axis measures each patent's cumulative citation score, and the horizontal axis measures each patent's rank within the

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[165] Morgan, *supra* note 119, at 281. One might also measure inequality in a distribution using the 90/10 ratio, which we have explained and reported in Part V.E *supra*. *See, e.g.*, Jonathan Fisher & Timothy M. Smeeding, *Income Inequality*, Pathways (Special Issue), 2016, at 32, 34, http://inequality.stanford.edu/sites/default/files/Pathways-SOTU-2016.pdf. However, because the 90/10 ratio examines only the extreme ends of a distribution, it does not offer a precise measure of inequality toward the center of a distribution. Nonetheless, one can use the 90/10 ratio as a robustness check for the Gini coefficient.

[166] *See* Morgan, *supra* note 119, at 281.

[167] *See id.*

LRDIMM standard, on the basis of its own citation score. The Gini coefficient is equal to $A \div (A + B)$, where A represents the red area between the 45-degree line and the Lorenz curve, B represents the gray area below the Lorenz curve, and $A + B$ represents the total area under the 45-degree line. Economists and statisticians employ various methodologies to estimate the Gini coefficient.[168] One commonly used methodology is the trapezoid-rule methodology,[169] which relies on the trapezoid rule in calculus for approximating the area under a curve.[170]

Figure A2. The Trapezoid-Rule Methodology
for Estimating the Gini Coefficient

Source: Authors' original figure.
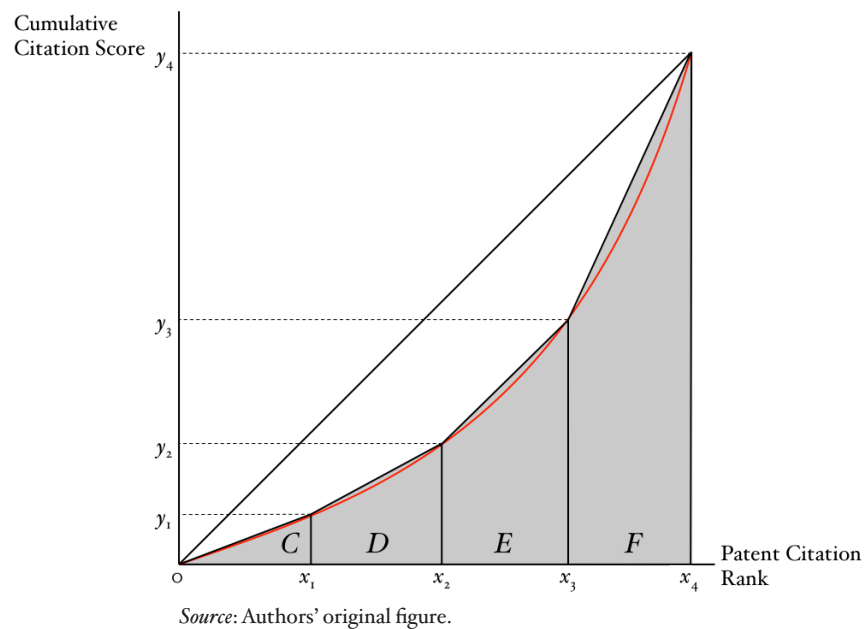
The trapezoid-rule methodology estimates the Gini coefficient "by applying the trapezoid rule to find the area under the piecewise linear Lorenz curve."[171] For example, in Figure A2, one can use the trapezoid-rule methodology to calculate an approximation of the area under the Lorenz curve by taking the sum of the areas of the trapezoids—that is, $C + D + E + F$. By

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[168] *See, e.g., id.* (showing the trapezoid-rule methodology for approximating the Gini coefficient); Gastwirth, *supra* note 119, at 309; Frank A. Farris, *The Gini Index and Measures of Inequality*, 117 Am. Mathematical Monthly 851, 852 (2010) (showing that, given an equation for the Lorenz curve, one can use integrals to compute the area between two curves); John Golden, *A Simple Geometric Approach to Approximating the Gini Coefficient*, 39 J. Econ. Educ. 68, 70–74 (2008) (proposing the *Z*-gradient rule, which uses numerical quintile data to estimate the Gini coefficient).

[169] *See, e.g.*, Morgan, *supra* note 119, at 281; Gastwirth, *supra* note 119, at 309; Farris, *supra* note 168, at 856; Golden, *supra* note 168, at 74–75.

[170] *See, e.g.*, Gilbert Strang, Calculus 222–23 (Wellesley-Cambridge Press 1991).

[171] Farris, *supra* note 168, at 857.

subtracting that area from the area under the 45-degree line—which is equal to the area of the triangle with vertices $(0, 0)$, $(x_4, 0)$, and (x_4, y_4)[172]—one can approximate the area between the 45-degree line and the Lorenz curve. Equation A1 summarizes the trapezoid-rule methodology:

$$G \approx (A_{45°} - \Sigma A_{\text{trapezoid}}) \div A_{45°}, \qquad\qquad \text{(A1)}$$

where G is the Gini coefficient, $A_{45°}$ is the area under the 45-degree line, and $\Sigma A_{\text{trapezoid}}$ is the sum of the areas of the trapezoids. Using the trapezoid-rule methodology, we obtained Gini coefficients of 0.33 when including self-citations and 0.49 when excluding self-citations for the 50 LRDIMM patents in our estimation sample.[173]

It is worth noting that the trapezoid-rule methodology underestimates the Gini coefficient because the Lorenz curve is convex.[174] Put differently, because the smooth Lorenz curve necessarily lies below the piecewise-linear approximation of the Lorenz curve defined by the sides of the trapezoids, as the red line in Figure A2 shows, the trapezoid-rule methodology overestimates the area below the Lorenz curve. However, as the number of trapezoids increases, the piecewise-linear approximation of the Lorenz curve will better approximate the shape of the smooth Lorenz curve, which will increase the accuracy of the estimation of the Gini coefficient. We use an alternative (and simpler) methodology, which Figure A3 demonstrates.

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[172] The equation for computing the area of a triangle is of course $0.5 \times b \times h$, where b is the base of the triangle and h is the height of the triangle. Thus, using simple rules of geometry, one can calculate the area below the 45-degree line in Figure A2 as $0.5 \times x_4 \times y_4$.

[173] We estimated the Gini coefficients using the Stata statistical software.

[174] *See, e.g.*, Gastwirth, *supra* note 119, at 309; Golden, *supra* note 168, at 74.

Figure A3. A Simple Alternative Methodology
for Estimating the Gini Coefficient
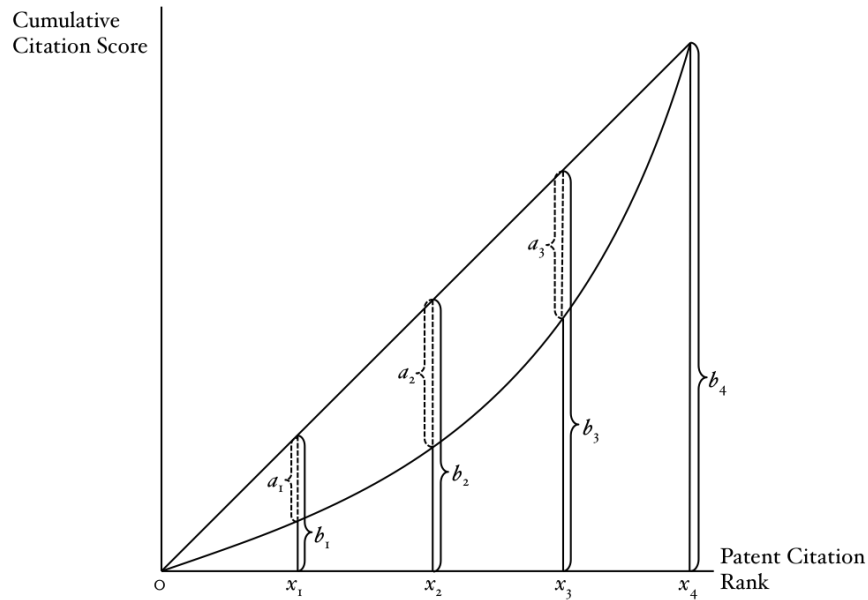
Source: Authors' original figure.

In Figure A3, a_1, a_2, and a_3 represent the difference (or vertical distance) between the 45-degree line and the Lorenz curve for patents with citation ranks x_1, x_2, and x_3, respectively. Similarly, b_1, b_2, b_3, and b_4 represent the difference (or vertical distance) between the x-axis and the 45-degree line for patents with citation ranks x_1, x_2, x_3, and x_4, respectively. (Figure A3 does not show a_4 because it is equal to zero.) To estimate the Gini coefficient, we divided the sum of the differences between the 45-degree line and the Lorenz curve for each patent by the sum of the differences between the x-axis and the 45-degree line for each patent, as Equation A2 shows:

$$G \approx \Sigma\, a_i \div \Sigma\, b_i, \tag{A2}$$

where $\Sigma\, a_i$ is the sum of the differences between the 45-degree line and the Lorenz curve for each patent and $\Sigma\, b_i$ is the sum of the differences between the x-axis and the 45-degree line for each patent. Our estimated Gini coefficients for the LRDIMM SEPs are 0.34 when including self-citations and 0.49 when excluding self-citations.[175] Thus, our estimations of the Gini coefficient are slightly higher than those we obtain using the trapezoid-rule methodology, both when including and excluding self-citations.

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[175] We estimated the Gini coefficients using the Stata statistical software.

Appendix III. Factual Summary of Patents Included in the Forward-Citation Analysis

Table A1. Active Patents Declared to Be Essential to JEDEC's LRDIMM Standard

Patent Number	Patent Rank	Current Assignee	Patent Title	Number of Citations	Patent-Citation Score	Patent Issue Date
6498766	28	Samsung Electronics Co. Ltd.	Integrated circuit memory devices that utilize indication signals to increase reliability of reading and writing operations and methods of operating same	95	71.5	2002-12-24
6980021	25	Inphi Corp.	Output buffer with time varying source impedance for driving capacitively-terminated transmission lines	81	3.47	2005-12-27
7058776	27	Samsung Electronics Co. Ltd.	Asynchronous memory using source synchronous transfer and system employing the same	77	0.275	2006-06-06
7107415	37	Round Rock Research LLC	Posted write buffers and methods of posting write requests in memory modules	29	25.1	2006-09-12
7120743	26	Round Rock Research LLC	Arbitration system and method for memory responses in a hub-based memory system	61	33.0	2006-10-10
7133972	4	Round Rock Research LLC	Memory hub with internal cache and/or memory access prediction	106	30.2	2006-11-07
7200062	23	Round Rock Research LLC	Method and system for reducing the peak current in refreshing dynamic random access memory devices	78	14.2	2007-04-03
7210059	7	Round Rock Research LLC	System and method for on-board diagnostics of memory modules	101	31.1	2007-04-24
7222213	32	Micron Technology Inc.	System and method for communicating the synchronization status of memory modules during initialization of the memory modules	39	57.4	2007-05-22

7234070	43	Round Rock Research LLC	System and method for using a learning sequence to establish communications on a high-speed nonsynchronous interface in the absence of clock forwarding	17	35.7	2007-06-19
7254036	19	Netlist Inc.	High density memory module using stacked printed circuit boards	95	48.6	2007-08-07
7260685	31	Round Rock Research LLC	Memory hub and access method having internal prefetch buffers	38	21.7	2007-08-21
7269042	13	Round Rock Research LLC	Memory stacking system and method	88	9.96	2007-09-11
7286436	12	Netlist Inc.	High-density memory module utilizing low-density memory components	106	39.3	2007-10-23
7289386	6	Netlist Inc.	Memory module decoder	117	23.2	2007-10-30
7307863	22	Rambus Inc.	Programmable strength output buffer for RDIMM address register	77	42.9	2007-12-11
7310752	49	Round Rock Research LLC	System and method for on-board timing margin testing of memory modules	7	44.8	2007-12-18
7375970	41	Netlist Inc.	High density memory module using stacked printed circuit boards	31	52.5	2008-05-20
7379316	5	Google Inc.	Methods and apparatus of stacking DRAMs	105	36.2	2008-05-27
7386656	18	Google Inc.	Interface circuit system and method for performing power management operations in conjunction with only a portion of a memory circuit	79	4.22	2008-06-10
7392338	21	Google Inc.	Interface circuit system and method for autonomously performing power management operations in conjunction with a plurality of memory circuits	73	12.0	2008-06-24
7408393	24	Inphi Corp.	Master-slave flip-flop and clocking scheme	65	53.6	2008-08-05
7472220	17	Google Inc.	Interface circuit system and method for performing power management operations utilizing power management signals	76	39.5	2008-12-30

7515453	20	Google Inc.	Integrated memory core and memory interface circuit	67	36.3	2009-04-07
7532537	2	Netlist Inc.	Memory module with a circuit providing load isolation and memory domain translation	124	34.4	2009-05-12
7580312	15	Google Inc.	Power saving system and method for use with a plurality of memory circuits	75	40.2	2009-08-25
7581127	16	Google Inc.	Interface circuit system and method for performing power saving operations during a command-related latency	74	37.8	2009-08-25
7590796	14	Google Inc.	System and method for power management in memory systems	76	67.3	2009-09-15
7599205	9	Google Inc.	Methods and apparatus of stacking DRAMs	79	41.8	2009-10-06
7619912	3	Netlist Inc.	Memory module decoder	109	40.8	2009-11-17
7630202	42	Netlist Inc.	High density module having at least two substrates and at least one thermally conductive layer therebetween	21	42.3	2009-12-08
7636274	29	Netlist Inc.	Memory module with a circuit providing load isolation and memory domain translation	47	45.0	2009-12-22
7724589	10	Google Inc.	System and method for delaying a signal communicated from a system to at least one of a plurality of memory circuits	74	60.3	2010-05-25
7730338	11	Google Inc.	Interface circuit system and method for autonomously performing power management operations in conjunction with a plurality of memory circuits	73	11.8	2010-06-01
7745919	45	Longitude Licensing Ltd.	Semiconductor device including a plurality of semiconductor chips and a plurality of through-line groups	13	24.7	2010-06-29

7761724	8	Google Inc.	Interface circuit system and method for performing power management operations in conjunction with only a portion of a memory circuit	72	44.9	2010-07-20
7916574	34	Netlist Inc.	Circuit providing load isolation and memory domain translation for memory module	33	44.8	2011-03-29
7952201	46	Longitude Licensing Ltd.	Semiconductor device including stacked semiconductor chips	8	7.41	2011-05-31
8001434	30	Netlist Inc.	Memory board with self-testing capability	43	45.0	2011-08-16
8072837	40	Netlist Inc.	Circuit providing load isolation and memory domain translation for memory module	23	17.3	2011-12-06
8081535	38	Netlist Inc.	Circuit for providing chip-select signals to a plurality of ranks of a DDR memory module	23	4.73	2011-12-20
8081536	35	Netlist Inc.	Circuit for memory module	30	24.0	2011-12-20
8081537	39	Netlist Inc.	Circuit for providing chip-select signals to a plurality of ranks of a DDR memory module	23	12.5	2011-12-20
8239812	51	Longitude Licensing Ltd.	Semiconductor device, adjustment method thereof and data processing system	3	12.6	2012-08-07
8264903	33	Netlist Inc.	Systems and methods for refreshing a memory module	28	16.8	2012-09-11
8381157	48	Longitude Licensing Ltd.	Semiconductor device, adjustment method thereof and data processing system	5	12.6	2013-02-19
8422263	47	Longitude Licensing Ltd.	Load reduced memory module and memory system including the same	7	2.17	2013-04-16
8473653	44	Longitude Licensing Ltd.	Semiconductor device, control method for the semiconductor device and information processing system including the same	12	17.5	2013-06-25

| 8489837 | 36 | Netlist Inc. | Systems and methods for handshaking with a memory module | 21 | 4.60 | 2013-07-16 |
| 9128632 | 52 | Netlist Inc. | Memory module with distributed data buffers and method of operation | 1 | 4.72 | 2015-09-08 |

Note: The number of citations and the patent-citation score are calculated including self-citations.

Table A2. Expired Patents Declared to Be Essential to JEDEC's LRDIMM Standard

Patent Number	Patent Rank	Current Assignee	Patent Title	Number of Citations	Patent-Citation Score	Patent Issue Date
6820181	1	Round Rock Research LLC	Method and system for controlling memory accesses to memory modules having a memory hub architecture	144	71.5	2004-11-16
7349277	50	Round Rock Research LLC	Method and system for reducing the peak current in refreshing dynamic random access memory devices	5	3.47	2008-03-25
7366920	53	Micron Technology Inc.	System and method for selective memory module power management	0	0.0275	2008-04-29

Note: The number of citations and the patent-citation score are calculated including self-citations.