Hedonic Prices, Patent Royalties, and the Theory of Value and Distribution: A Comment on Sidak and Skog

Alexander Galetovic*

In Hedonic Prices and Patent Royalties, J. Gregory Sidak and Jeremy O. Skog use a hedonic regression to estimate the incremental cumulative value of a bundle of standard-essential patents (SEPs) that define Joint Electron Devices Engineering Council’s (JEDEC’s) load-reduced dual-inline memory module (LRDIMM) standard. They then apportion the estimated cumulative value pro rata to the adjusted number of forward citations received by each SEP.

The motivation for the Sidak-Skog hedonic regression analysis is to answer a practical question that courts have been asked several times: How much can a patent holder charge for an SEP that it has agreed to license on fair, reasonable, and nondiscriminatory (FRAND) terms? In some cases, a court must act when implementer I sues patent holder H claiming that H violated its FRAND commitment by overcharging for its SEPs. In other cases, patent holder H sues implementer I claiming that I has infringed H’s patents. In both cases, the court must determine the FRAND royalty that H may charge.

Behind any valuation and apportionment exercise lies an underlying theory of value and distribution—a theory that explains where the value of the patents comes from and how that value is, or should be, distributed among consumers and the owners of the factors of production. This

* Professor of Economics, Universidad de los Andes, Chile; Visiting Fellow, Hoover Institution. Email: agaletovic@gmail.com. I thank Stephen Haber, Gregory Sidak, and Jeremy Skog for helpful discussions and comments. I am grateful for the financial support of Hoover IP. The views expressed in this article are solely my own. Copyright 2018 by Alexander Galetovic. All rights reserved.


comment therefore asks: What is the theory of value and distribution underlying Sidak’s and Skog’s method?

I. The Incremental Value of a Bundle of Technologies and the Hedonic Method

The first part of Sidak’s and Skog’s method estimates the value of the technology with a hedonic regression. In this part, I explain that this is a rigorous theory of value that can be implemented empirically to gauge the value of a technology embedded in a standard.

A. The Incremental Value of a Bundle of Technologies

The hedonic method relates a good’s characteristics with consumers’ willingness to pay. A good that implements a new technology (in this case, a memory module that complies with the LRDIMM standard) has more characteristics that consumers value (for example, speed and capacity) and are willing to pay for. Consequently, one can compare what consumers pay in the market for the new technology versus the old technology and estimate the consumer’s differential willingness to pay for the new technology.

To see how the hedonic method works, assume that modules embedding two different technologies, old (O) and new (N), are available in the market. In Figure 1, the vertical axis measures consumers’ willingness to pay for an old and a new module. To simplify, assume that all consumers are willing to pay up to \( v_O \) to use the old module (the horizontal, flat line). At the same time, all consumers agree that new modules are better, and are willing to pay more than \( v_O \) for them. Nevertheless, some consumers are willing to pay more than others for the new module. Thus, the horizontal axis graphs the differential (\( \Delta \)) willingness to pay for a module embedding the new technology; the upward-sloping line measures the differential, which varies between \( 0 \) and \( v_N - v_O \), as some consumers value new modules more.
In a market equilibrium, some consumers will use the new module and some will stick with the old one, a key fact that is exploited by the hedonic method. To see why, assume that in the observed market equilibrium, implementers charge $p_N > p_O$ for a new module so that the new module is strictly more expensive than the old module. A consumer will choose the module that maximizes her consumer surplus. If she buys the old module, her consumer surplus is $v_O - p_O$ per module—the difference between her willingness to pay and the module’s price. If, on the other hand, she buys a new module, her consumer surplus is $v_O + \Delta - p_N$, meaning that consumer surplus grows with $\Delta$ (as some consumers value the new technology more than others), but decreases with the new module’s price. Therefore, a consumer will be indifferent between purchasing a new and an old module only if she obtains the same consumer surplus, namely:

$$v_O + \Delta - p_N = v_O - p_O,$$

or

$$\Delta = p_N - p_O,$$

(i)
where $\Delta$ denotes the differential valuation of the consumer who is indifferent between the old and the new module.

As can be seen in Figure 1, consumers with a differential willingness to pay $\Delta > \Delta$ obtain greater consumer surplus using the new module, and consumers with $\Delta < \Delta$ are better off buying the old module. For the indifferent consumer, the differential valuation exactly compensates the new module's higher price. The hedonic regression uses the observed difference from Equation 1 to estimate the differential value of the new technology by regressing $\Delta$ against the characteristics of the technology, which drive consumers' differential willingness to pay.

Three implications follow. First, as Equation 1 shows, the hedonic method estimates the differential willingness to pay of the marginal consumer—the consumer who is indifferent between the new and old technology. Most economists would agree that this is a sensible way of estimating the incremental value of a technology. Indeed, for many years, statistical government offices, such as the Bureau of Labor Statistics, have used hedonic regressions to distinguish price increases wrought by quality improvements from price increases caused by inflation. \(^3\)

Second, note that many consumers value the new module's differential characteristics by more than $v_o + \Delta$ and would be willing to pay more than the marginal consumer (represented by the shaded triangle in Figure 1). A hedonic regression will not capture this additional value created by the new module, however, because it estimates only the differential willingness to pay at the margin by the indifferent consumer. Consequently, the total value created by the new module will be, in general, larger than the estimate from the hedonic regression.

Third, note that the price differential $\Delta = p_N - p_O$ reflects the market's equilibrium—the outcome of the interaction between supply and demand. Consequently, the observed price difference also depends on supply-side factors, such as the relative costs of manufacturing old and new modules, implementer market structure, the intensity of price competition, and the royalties that the owners of IP charge. Supply conditions change over time, causing the differential value at the margin to change too, but hedonic theory does not say anything about the supply side of the market.

B. Hedonic Regressions and the Incremental Value of JEDEC's LRDIMM Standard

The key observation in Sidak's and Skog's article is that the equilibrium differential willingness to pay for a module is systematically related to the module's characteristics. If products embedding different characteristics are

available in the market, researchers can use variation in module prices to estimate how the marginal consumers’ demonstrated willingness to pay varies with module characteristics. Moreover, if characteristics are systematically related to technologies and SEPs, one can use the estimated regression to estimate the incremental value they add at the margin.

Using this reasoning, Sidak and Skog run the following regression:

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\text{Price} = \alpha + \beta_1 \cdot \text{LRDIMM} + \beta_2 \cdot \text{DDR4} + \beta_3 \cdot \text{GB} + \beta_4 \cdot \text{year}. \tag{2}
\]

In this regression, LRDIMM and fourth-generation double data rate (DDR4) are dummy variables, and a module embedding the technologies in registered dual-inline memory modules (RDIMM) and third-generation double data rate (DDR3) is the base category. The estimation yields \(\alpha = \$46.67\): on average, a module embedding the technologies in RDIMM and DDR3 sells for a base price of \$46.67 (augmented by its size in gigabytes (GB) and varying with the year). Moreover, \(\beta_1 = \$100.47\): embedding the technology in LRDIMM adds \$100.47 to the module. Last, \(\beta_2 = \$48.75\): going from DDR3 to DDR4 adds, on average, an additional \$48.75 to a consumer’s willingness to pay at the margin. Hence, the differential willingness to pay for a module that complies with the LRDIMM standard is \$100.47.

Why are modules embedding the technologies in LRDIMM and DDR4 more valuable? It turns out that these modules can handle dynamic random-access memory (DRAM) faster for any given capacity, and the regression says that the marginal consumer is willing to pay more for this attribute. Note, however, that the hedonic regression that Sidak and Skog use does not directly estimate the value of speed. Indeed, the only characteristic that appears in Regression 2 that consumers value directly is capacity, which is measured in GB.

Sidak and Skog also argue that parameter \(\alpha\) measures the value of standardization itself. They reason that SEP owners and implementers realized the value of standardization when they developed the RDIMM standard. Hence, they conclude that a hedonic regression separates the differential value created by new standards (in this case LRDIMM and DDR4) from the value of standardization itself. This conclusion would be important because courts have claimed that SEP holders are not entitled to the value of standardization itself.

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4 Id.
5 That is, the dummy variable LRDIMM is equal to 1 if the module embeds the technology in the LRDIMM standard and zero if it only embeds the RDIMM standard. Similarly, the dummy variable DDR4 is equal to 1 if the module embeds the technology in the DDR4 standard, and zero if it only embeds the DDR3 standard.
While using parameter $\alpha$ as an estimate of the value of standardization is a practical way of implementing what courts have asked experts to distinguish, as a matter of economics it is doubtful that $\alpha$ is the value of standardization itself. As I explained with Stephen Haber in The Fallacies of Patent Holdup Theory, a standard is an input, and inputs are not valuable in and of themselves.\(^6\) On the contrary, inputs have value only if consumers obtain utility from the final product they produce (in this case, fast memory access). Therefore, the demand for standardization is a derived demand, and it only pays to standardize technologies that do things that are valued by consumers. In other words, the value of a standard stems from the things consumers can do with the goods that embed the standardized technologies.\(^7\)

In this case, standardization is a valuable input of production because it reduces the cost of using technologies embedded in the RDIMM and DDR3 standards. Furthermore, standardization reduces the cost of using technologies embedded in the LRDIMM and the DDR4 standards. The gains from standardization were worked out anew in standard-development organizations; otherwise, it would not have been necessary to develop a new standard for LRDIMM and DDR4. In other words, the value of standardization is not inherited by the next technological generation. Rather, standards must be reworked anew by every technological generation.

In addition, as discussed above, a hedonic regression relates characteristics to differential willingness to pay, as reflected in market prices. Therefore, the coefficient $\alpha$ merely indicates that the marginal consumer was willing to pay $46.67 for the characteristics embedded in the technologies in RDIMM and DDR3. The coefficient neither links this value to inputs nor to “standardization.”

Regardless of the interpretation of the coefficient $\alpha$, however, a regression like Equation 2 has the important feature that differential willingness to pay is estimated from observed market transactions—not from a hypothetical exercise in which the actual technology competes with a hypothetical technology that never actually existed. While Equation 2 cannot explain why consumers put a higher value on modules that comply with the LRDIMM and DDR4 standards, Equation 2 answers an important question: How much more are consumers willing to pay for modules that embed the latest technologies, given that they still have the option to use older technologies? It would be difficult to exaggerate the difference between, on the one hand, Sidak’s

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and Skog’s approach, which deduces the value of a technology from observed market transactions, and, on the other hand, valuation approaches that are based on unobserved hypotheticals. More broadly, a proxy of the value of a technology deduced from market transactions imposes a quantitative benchmark and a reality check on any valuation method that courts may use. For these reasons, Sidak’s and Skog’s approach is a significant step forward in the right direction.

At the same time, as an estimate of how much SEP holders should receive in royalties, a hedonic regression like Equation 2 has at least two limitations. One is that even in a thick market, such as real estate, different algorithms developed by different analysts to estimate the value of characteristics typically produce substantially different outcomes that in turn differ from the actual market prices that people obtain when they trade. Therefore, although hedonic regressions produce insights that are important in understanding how markets work, it is less clear that they can substitute for market-determined royalties, which value technologies directly.

The second limitation of a hedonic regression like Equation 2 is that, as discussed above, it does not provide a theory of distribution of the differential value created by the new technology among factors of production. That would require a theory of the supply side of the market, which links goods’ prices and factor remuneration. Therefore, a hedonic regression cannot fully answer the key question that courts are asked: How much should a given SEP owner receive for her patents?

Sidak and Skog are aware of this deficiency. Before apportioning the incremental value of the LRDIMM standard among SEP holders, they subtract the incremental cost of manufacturing such a module—which they assumed to be equal to $20.70—from the estimated differential willingness to pay for LRDIMM-compliant modules ($100.47). Sidak and Skog conclude that the value to be apportioned among SEPs equals the difference between differential willingness to pay for the technology and the cost of implementing it. Again, Sidak and Skog are operating within the legal constraints of existing judicially created doctrine. As a matter of economics, however, this may be controversial, for part of that value at the margin may be created by other factors of production, such as those provided by implementers. Economics can determine what this equilibrium distribution should be, but only after one posits a theory of market equilibrium determination on the supply side of the market.

II. Forward Citations to Apportion Value

The lack of a theory of the supply side of the market leads to the second part of the article: a method to apportion the value created by LRDIMM among
SEPs. I explain in this part that available apportionment methods are a practical way of answering a question in the terms that courts consider relevant and admissible. Nevertheless, they are devoid of economic content and are not based on a rigorous theory of distribution.

A. Sidak’s and Skog’s Apportionment Method

As explained above, Sidak and Skog estimate that the incremental value of the SEPs that read on the LRDIMM standard is $79.77 (that is, $100.47 – $20.70). They apportion that value among each of the SEPs that read on the LRDIMM standard with a time-count and citation-count weighted index of SEPs.

Apportionment proceeds in three steps. First, the researcher identifies the universe of active patents that patent holders have declared essential. Second, the researcher counts the number of forward citations received by each SEP. Third, the royalty is determined to be equal to the incremental value of the SEPs that read on the LRDIMM standard ($79.77) times the patent’s share of the total number of citations. For example, Sidak and Skog report that Netlist’s SEPs account for 42.96 percent of the forward citations received by SEPs declared to the LRDIMM standard: “The value of the LRDIMM standard attributable to Netlist’s LRDIMM SEP portfolio is therefore $79.77 x 42.96 percent = $34.27.”

B. Can We Trust Apportionment Exercises?

There are reasons, however, to be skeptical of any apportionment exercise. To begin, the definition of an SEP and the number of patents that are “really” essential is contentious. For example, Keith Mallinson compared two studies commissioned by industry participants that purported to count the number of essential patent families owned by major SEP holders in mobile phones. One study claimed to have relied on “industry experts that included physics PhDs, wireless engineers, patent legal specialists, and former patent office employees.” The other study claimed to have accumulated six years of experience assessing essentiality. If determining essentiality were an exact science, both studies should have allocated the same number and share of SEPs to each patent holder, and a plot of the data in a two-dimensional graph

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8 Sidak and Skog note that self-citations should not be counted. Also, they adjust the rough share in the total number of forward citations by the age of the patent, because older patents have received more forward citations at the time of any study, as they have been around for longer.

9 Sidak & Skog, supra note 1, at 653.


should have accumulated data points on a 45-degree line, as demonstrated in Figure 2.

By contrast, Mallinson found that the correlation between both studies was exactly zero. Mallinson added six more studies to his original two, and he found that the correlation between pairs of studies was most of the time below 0.5 when the studies were conducted by different assessors. He concluded that studies of essentiality produce widely differing results.

Next, as Mallinson explains, it is costly to map each patent into the respective standard. For example, a study commissioned by the European Commission estimates the following costs of establishing essentiality:

(i) Approx. 600–1,800 Euro per patent (1–3 days of work) for a first instance essentiality test performed by the SSO internally, with the confidence level appropriate for patent disclosure obligations at an SSO. (The level is often lower, as a patent in the same patent family will need fewer individual resources and because firms may possess previous information on their patents);

(ii) Approx. 5,000–15,000 Euro per patent for an essentiality test performed by a third party in the context of a patent pool. The lower boundary fee

14 Id.
assumes that prior information from the patent owner is available and only up to three patent claims (selected by the owner) are tested; and

((3) Approx. >20,000 Euro per patent for an extensive essentiality and/or infringement test in the context of a court case, including extensive search for technologies that may constitute alternative solutions.  

It seems, therefore, that counting SEPs is expensive and unreliable. Unsurprisingly, when market participants bargain for royalties they do something different: they agree on royalties to license an entire portfolio, without distinguishing between SEPs and non-SEPs, and without doing a patent-by-patent assessment.

In addition, the claim that a patent’s share in a citation-weighted index is a good proxy for its share in the aggregate market royalty does rely on empirical evidence. Nevertheless, while there are a few empirical articles that link forward citations with different measures of patent value, these studies are rather tentative.  Moreover, the empirical link between forward citations and patent value is not always tight. For example, in a well-known study, R’s tend to be of the order of 0.1 to 0.3, such that predicted values are rather noisy.  Thus, forward citations may be suitable to estimate the value of large portfolios, but it is less likely that forward citations will yield accurate estimates of the value of individual patents or small portfolios.

The main limitation of forward citations is that there is no theory explaining the link between the number of forward citations and the value created by the technology claimed by the SEP. Indeed, Sidak’s and Skog’s apportionment rule is slightly at odds with their hedonic regression. If differential willingness to pay stems from the features that consumers value, then the value created by an SEP depends on its useful, novel, and nonobvious features, which are described in the SEP’s claims. By contrast, as Ron Katznelson explains, a patent is cited subsequently because of the information it contains in its disclosure and teaching, and in order to limit the scope

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16 See, e.g., Manuel Trajtenberg, A Penny for Your Quotes: Patent Citations and the Value of Innovations, 21 RAND J. Econ. 172 (1990) (discussing the relation between citation counts and social surplus created by a patented technology); Jesse Micah Giannino, An Empirical Examination of the Value of Patented Inventions Using German Employee Inventors’ Compensation Records (Fall 2003) (unpublished Ph.D. dissertation, University of California, Berkeley) (on file with ProQuest Dissertations Publishing) (finding that the royalties received by inventors/patent holders at nine major German corporations under the German Employee Compensation Act of 1957 correlated positively with the number of forward citations received by the patent); Bronwyn H. Hall, Adam Jaffe & Manuel Trajtenberg, Market Value and Patent Citations, 36 RAND J. Econ. 16 (2005) (discussing the relation between the number of forward citations and a firm’s stock market value).
17 Hall, Jaffe & Trajtenberg, supra note 16.
of the claims of the citing patent. It is not clear what links, on the one hand, the value created by the SEP’s claims, and, on the other hand, the narrowing down of the claims of the subsequent patents. Thus, although forward citations are useful to learn about how much value patents create, it seems that we have to learn a lot more before citations become a reliable tool to apportion royalties.

Last, even if an empirical link between forward citations and patent value could be established, forward citations would still not provide a theory of the supply side of the market. For this reason, forward citations cannot tell apart the value created by the SEP or SEP portfolio from the value created at the margin by other inputs of production. For that analysis, a theory of the supply side of the market is required.

C. When Is Top-Down Apportionment Appropriate?

One might be tempted to classify Sidak’s and Skog’s valuation and apportionment method as a variant of the so-called top-down approach. As Gregory Leonard and Mario Lopez explain, under a top-down approach, the researcher first fixes the aggregate royalty, then divides it among SEPs according to some distributional criterion. When is such an approach appropriate?

Experts design measurement methods limited by legal constraints. As a matter of economics, however, the main shortcoming of the top-down approach is that it is not based in any theory of value and distribution. Different authors have differing views about the determinants of the fair and reasonable aggregate royalty, and different authors use different apportionment rules. One could give some coherence to the top-down approach, however, by assuming that royalties are just a means to split a pie of fixed size—a sort of license to charge a private tax. It would then be the task of the courts or political institutions to set the aggregate royalty and apportion it among those who, according to some criterion, are entitled to a part of the pie. That method would just need to comply with the criterion underlying the right to tax. It need not (and most likely would not) be consistent with any economic theory of value and distribution.

The problem with assuming that royalties have nothing to do with what consumers value is that it ignores that firms invest in R&D with the expectation of contributing to products that have a market because consumers value them. If this expectation were taken into account, then both the value of the technologies and the rewards to the owners of intellectual property would

be endogenous variables determined in a market equilibrium. The top-down approach is oblivious to this fact.

Indeed, to a large extent, Sidak’s and Skog’s hedonic regression contradicts the view that patents are merely rights to tax. On the contrary, they show that the technologies embedded in standards are systematically related to consumers’ willingness to pay. Similarly, in a market equilibrium the distribution of the value among factors of production is related to their contribution at the margin. For this reason, apportionment methods should be based on a theory of the supply side of the market, not on ad hoc apportionment methods.

Conclusion

Behind Sidak’s and Skog’s hedonic price approach is an explicit and well-established theory of value: in a market’s equilibrium, differential prices of modules embedding different technologies are equal to consumers’ differential willingness to pay at the margin. Therefore, one can use observed price differences to estimate the value of the underlying technologies.

Sidak and Skog make an important contribution, because they link technology valuation with actual and observable market transactions and prices. This sets them apart from authors that argue that courts should compare the new technology with a hypothetical technology that never existed and was seemingly discarded by the standard-setting organization (SSO). In such a hypothetical exercise there would not be any role for economics or economists. Engineers or technologists would debate hypothetical alternatives to the real world. Moreover, theories that are directly linked with observable data are testable. By contrast, theories that rest on hypotheticals that never came to be are seldom testable and should be neither a guide to policy nor to royalty determination. Last, the hedonic method imposes a quantitative benchmark and reality check on any valuation method that courts may use.

At the same time, Sidak and Skog stop short of providing a theory of the supply side of the market. For this reason, though their apportionment method is a practical answer to an important question, circumscribed by both judicially created doctrine and the Federal Rules of Evidence, it is not a theory of distribution that can reliably estimate how much a patent holder and other factors of production would receive in a market’s equilibrium. In that sense, their method is still a work in progress.